Test 3 Review

Trigonometry & Vectors

Modules 10–13 | Sections 5.1, 5.2, 5.3, 7.1, 8.4

Math 130: Advanced Technical Mathematics | Unit 3

Test 3 — What to Expect

1	Given cos θ = adj/hyp, find sin θ and tan θ	M10 (5.2)
2	Sketch angle, find coterminal & reference angles (degrees + radians)	M10/11 (5.1/5.3)
3	Find two angles (0°–360°) with a given trig value (e.g. $\sin\theta = \sqrt{3}/2$)	M11 (5.3)
4	(a) Evaluate trig of ref angle (b) Trig values from point P(x, y)	M11 (5.3)
5	Exact side lengths using special triangles (30-60-90, 45-45-90)	M12 (7.1)
6	Solve a right triangle given two sides — find all angles & third side	M12 (7.1)
7	Area of a triangle: $A = \frac{1}{2}bc \sin A$	M11 (5.3)
8	Two-triangle elevation/depression word problem	M12 (7.1)
9	Vectors: component form, operations, magnitude, direction angle	M13 (8.4)

Reference sheet provides: $\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, Area = $\frac{1}{2}bc \sin A$

Formula Quick Reference

Angles	Coterminal: $\theta \pm 360°$ (or $\pm 2\pi$) | Arc length: $s = r\theta$ (θ in radians) | Convert: deg $\times \pi/180$ = rad
SOH CAH TOA	sin = opp/hyp | cos = adj/hyp | tan = opp/adj | Inverse: $\theta = \sin^{-1}$(ratio)
Reference Angles	Q I: $\theta' = \theta$ | Q II: $180° - \theta$ | Q III: $\theta - 180°$ | Q IV: $360° - \theta$
Special Triangles	30-60-90 → $1:\sqrt{3}:2$ | 45-45-90 → $1:1:\sqrt{2}$ | $\sin 30°=1/2$, $\sin 45°=\sqrt{2}/2$, $\sin 60°=\sqrt{3}/2$
ASTC Signs	Q I: All + | Q II: Sin + | Q III: Tan + | Q IV: Cos +
General Trig	$\sin\theta = y/r$ | $\cos\theta = x/r$ | $\tan\theta = y/x$ | $r = \sqrt{x^2+y^2}$
Triangle Area	$A = \frac{1}{2}bc \sin A$ (two sides + included angle, works for ANY triangle)
Vectors	$v = \langle x_2-x_1, y_2-y_1 \rangle$ | $\lvert v \rvert = \sqrt{a^2+b^2}$ | $\theta = \tan^{-1}(b/a)$ + quadrant adj.

Problem 1: Find sin θ and tan θ

Given $\cos \theta = 9/41$. Find $\sin \theta$ and $\tan \theta$.

$$\cos \theta = \text{adj}/\text{hyp} = 9/41$$

Find opp: $\text{opp}^2 = 41^2 - 9^2 = 1681 - 81 = 1600$

$$\text{opp} = \sqrt{1600} = 40$$

$$\sin \theta = \text{opp}/\text{hyp} = 40/41$$

$$\tan \theta = \text{opp}/\text{adj} = 40/9$$

Problem 2: Coterminal & Reference Angles

(a) $\theta = -97°$

Sketch: 97° clockwise → Q III

Positive coterminal:

$$-97° + 360° = 263°$$

Negative coterminal:

$$-97° - 360° = -457°$$

Reference angle:

$$263° \text{ in Q III} \rightarrow \theta' = 263° - 180° = 83°$$

(b) $\theta = 9\pi/7$

Sketch: $9\pi/7 \approx 231.4°$ → Q III

Positive coterminal:

$$9\pi/7 + 2\pi = 9\pi/7 + 14\pi/7 = 23\pi/7$$

Negative coterminal:

$$9\pi/7 - 2\pi = 9\pi/7 - 14\pi/7 = -5\pi/7$$

Reference angle:

$$\text{Q III} \rightarrow \theta' = 9\pi/7 - \pi = 9\pi/7 - 7\pi/7 = 2\pi/7$$

Problem 3: Find Angles from a Trig Value

Find two angles between 0° and 360° where $\sin \theta = \sqrt{3}/2$.

Step 1: Reference angle

$\sin \theta = \sqrt{3}/2 \rightarrow$ ref angle = 60° (from 30-60-90: $\sin 60° = \sqrt{3}/2$)

Step 2: Which quadrants is sin positive?

sin > 0 in Q I and Q II (ASTC: "All" and "Sine")

Step 3: Apply reference angle formulas

Q I: $\theta = 60°$

Q II: $\theta = 180° - 60° = 120°$

Problem 4: Trig Values

(a) Evaluate $\sin 240°$

Ref angle: $240° - 180° = 60°$

Q III → sin is negative

$$\sin 240° = -\sin 60° = -\sqrt{3}/2$$

(b) Trig values from $P(-2, -5)$

$x = -2$, $y = -5$ → Q III

$$r = \sqrt{(4 + 25)} = \sqrt{29}$$

$\sin \theta = -5/\sqrt{29} = -5\sqrt{29}/29$

$\cos \theta = -2/\sqrt{29} = -2\sqrt{29}/29$

$\tan \theta = (-5)/(-2) = 5/2$

Q III check: sin −, cos −, tan + ✓

Problem 5: Exact Values (Special Triangles)

Use exact trig values ($\sqrt{2}/2$, $\sqrt{3}/2$, $\sqrt{3}$, etc.) — NO decimal approximations!

30-60-90: opp 30° = 17, find others

Side 17 is opp 30° (short leg)

hyp = 2 × short leg = 2 × 17 = 34

long leg = short × $\sqrt{3}$ = $17\sqrt{3}$

Ratios: 1 : $\sqrt{3}$: 2

$\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, $\tan 30° = \sqrt{3}/3$

45-45-90: hyp = 24, find legs

hyp = leg × $\sqrt{2}$

24 = leg × $\sqrt{2}$

leg = $24/\sqrt{2} = 24\sqrt{2}/2 = 12\sqrt{2}$

Both legs equal: $x = y = 12\sqrt{2}$

Ratios: 1 : 1 : $\sqrt{2}$

$\sin 45° = \cos 45° = \sqrt{2}/2$, $\tan 45° = 1$

Problem 6: Solve a Right Triangle

Given $a = 11$, $c = 35$ (hypotenuse). Find A, B, and b.

$\sin A = a/c = 11/35 \approx 0.3143 \quad \rightarrow \quad A = \sin^{-1}(0.3143) \approx 18.3°$

$B = 90° - 18.3° = 71.7°$

$b = \sqrt{(c^2 - a^2)} = \sqrt{(1225 - 121)} = \sqrt{1104} \approx 33.2$

Solution: $A \approx 18.3°$, $B \approx 71.7°$, $C = 90°$, $a = 11$, $b \approx 33.2$, $c = 35$

Check: $11^2 + 33.2^2 = 121 + 1102.2 = 1223.2 \approx 35^2 = 1225$ ✓

Problem 7: Triangle Area (A = ½bc sin A)

Find the area: $A = 76°$, $b = 34$, $c = 21$.

Area = ½ × b × c × sin A

Area = ½ × 34 × 21 × sin 76°

Area = ½ × 714 × 0.9703

Area ≈ 346.4 square units

This formula is on the reference sheet. It works for ANY triangle — not just right triangles.

Problem 8: Two-Triangle Word Problem

Pattern: Two elevation angles to the same height, known distance between observation points.

Setup (let h = height, d = closer distance, gap between points):

1. Closer: tan(large angle) = h / d
2. Farther: tan(small angle) = h / (d + gap)

3. Both equal h → set right sides equal:

$$d \times \tan(\text{large}) = (d + \text{gap}) \times \tan(\text{small})$$

4. Solve for d: d[tan(large) − tan(small)] = gap × tan(small)

$$d = \text{gap} \times \tan(\text{small}) \,/\, [\tan(\text{large}) - \tan(\text{small})]$$

5. **Then: h = d × tan(large angle)**

Problem 9: Vectors

Component Form

P(3,−1) to Q(−2,4):

$\mathbf{v} = \langle -5, 5\rangle$

Operations

2u−3v where u=⟨2,−3⟩, v=⟨−1,4⟩:

$\langle 4,-6\rangle - \langle -3,12\rangle = \langle 7,-18\rangle$

Direction Angle

v = ⟨−5, 5⟩ (Q II):

$\theta = \tan^{-1}(5/-5)+180^\circ = 135^\circ$

Direction Angle Quadrant Adjustment: Q I: use as-is | Q II/III: add 180° | Q IV: add 360°

Magnitude: $|v| = \sqrt{a^2 + b^2}$ | **Scalar mult:** $k\langle a,b\rangle = \langle ka, kb\rangle$

Test Day Tips

Show all work — partial credit is available

Check calculator mode (DEGREE vs RADIAN) before every problem

Special triangles: leave answers exact ($\sqrt{3}$, $\sqrt{2}$) — don't round

ASTC: "All Students Take Calculus" for quadrant sign checks

Two-triangle problems: set up two tan equations with shared h, solve for d first

Vectors: always adjust direction angle for quadrant (Q II/III: $+180°$, Q IV: $+360°$)

Reference sheet gives: $\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, Area $= \frac{1}{2}bc \sin A$

Good luck on Test 3!