# Module 13

## Section 8.4

Vectors

# Why Vectors?

Many real-world quantities need both size AND direction.

- Velocity — wind at 15 mph to the north-east
- Force — 50 N pulling down at 30°
- Displacement — walking 3 blocks east, then 2 north

Scalars (speed, mass, temperature) need only size.

# Module 13 — Skills Overview

1. Writing a vector in component form from initial/terminal points
2. Vector addition and scalar multiplication: component form
3. Finding magnitude and direction of a vector from its graph
4. Finding the direction angle of a vector in ai + bj form
5. (Supplementary) Finding the components of a vector given magnitude and angle

# Vectors and Scalars

## Scalar

Magnitude only (a single number)

*Examples: speed, temperature, mass*

## Vector

Magnitude AND direction

*Examples: velocity, force, displacement*

## Notation

Component form: $v = \langle a, b \rangle = a\,i + b\,j$

Unit vectors: $i = \langle 1, 0 \rangle$, $j = \langle 0, 1 \rangle$

Zero vector: $0 = \langle 0, 0 \rangle$ (no magnitude, no direction)

Magnitude: $|v| = \sqrt{a^2 + b^2}$ Direction angle: $\theta$ (CCW from +x)

# Writing a Vector in Component Form

**Formula:** $v = \langle x_2 - x_1, y_2 - y_1 \rangle$ (terminal minus initial)

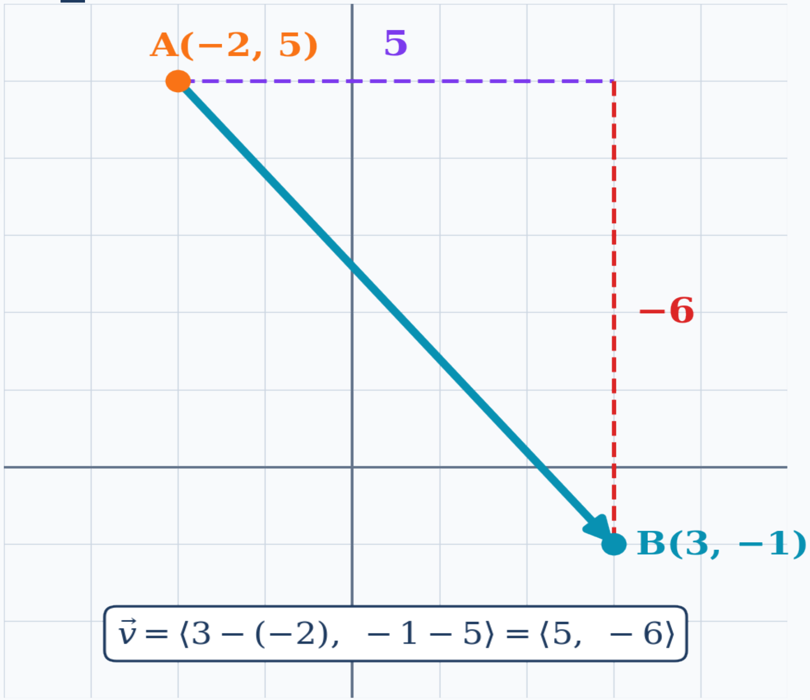
**Example: A(−2, 5) to B(3, −1)**

$v = \langle 3 - (-2), -1 - 5 \rangle$

$$\mathbf{v = \langle 5, -6 \rangle}$$

*Meaning: 5 units right, 6 units down*

Component form is position-independent — only the displacement matters.



# Vector Addition & Scalar Multiplication

**Addition** $\langle a, b\rangle + \langle c, d\rangle = \langle a+c, b+d\rangle$

**Subtraction** $\langle a, b\rangle - \langle c, d\rangle = \langle a-c, b-d\rangle$

**Scalar ×** $k\langle a, b\rangle = \langle ka, kb\rangle$

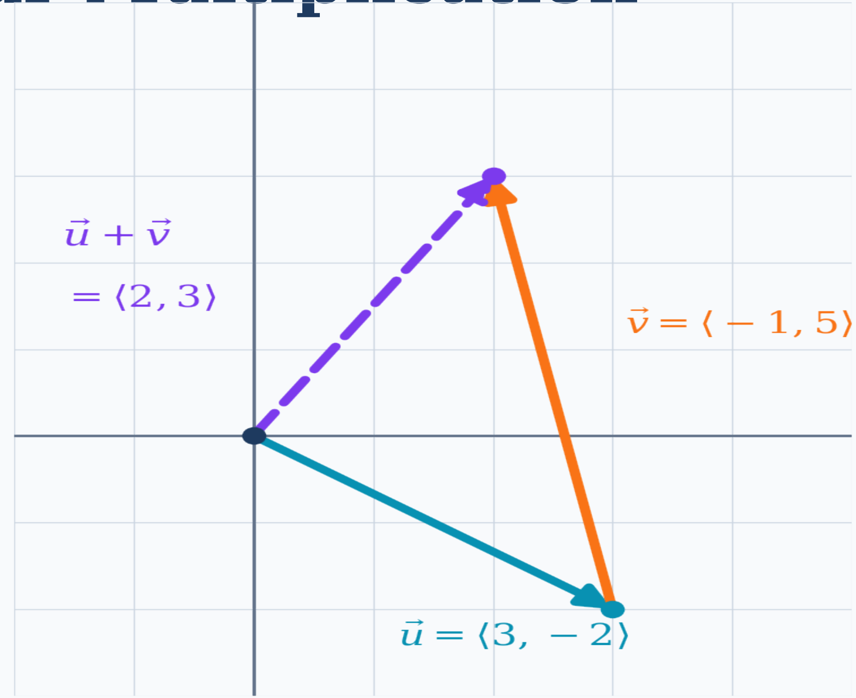
**$u = \langle 3, -2\rangle$ and $v = \langle -1, 5\rangle$**

**$u + v = \langle 3+(-1), -2+5\rangle = \langle 2, 3\rangle$**

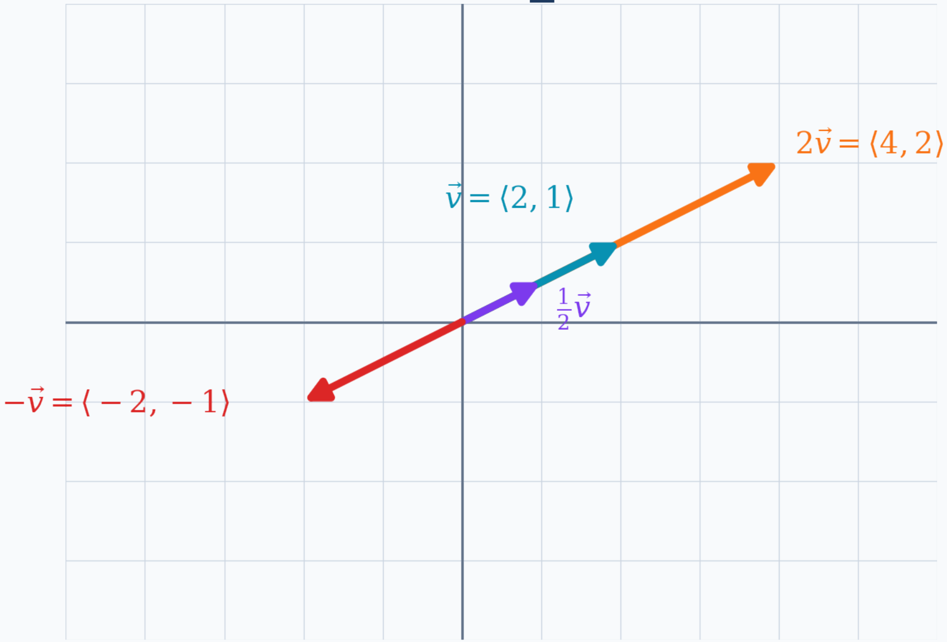
**$3u - 2v$:**

$3\langle 3,-2\rangle - 2\langle -1,5\rangle = \langle 9,-6\rangle - \langle -2,10\rangle$

**$= \langle 9-(-2), -6-10\rangle = \langle 11, -16\rangle$**



# Scalar Multiplication — Visual




## Rules

$|k| > 1$ → stretches the vector

$0 < |k| < 1$ → shrinks the vector

**$k < 0$ → reverses direction**

**$k\langle a, b\rangle = \langle ka, kb\rangle$**

**Magnitude: $|kv| = |k| \times |v|$**

# Magnitude and Direction

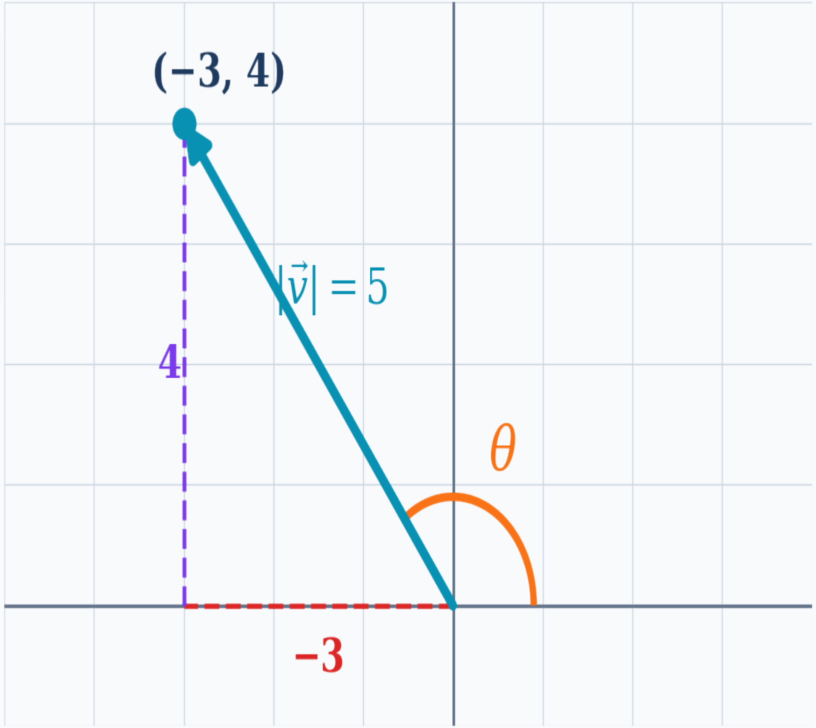
**Magnitude:** $|v| = \sqrt{(a^2 + b^2)}$

**Direction:** $\theta = \tan^{-1}(b / a)$
**(adjust for quadrant)**

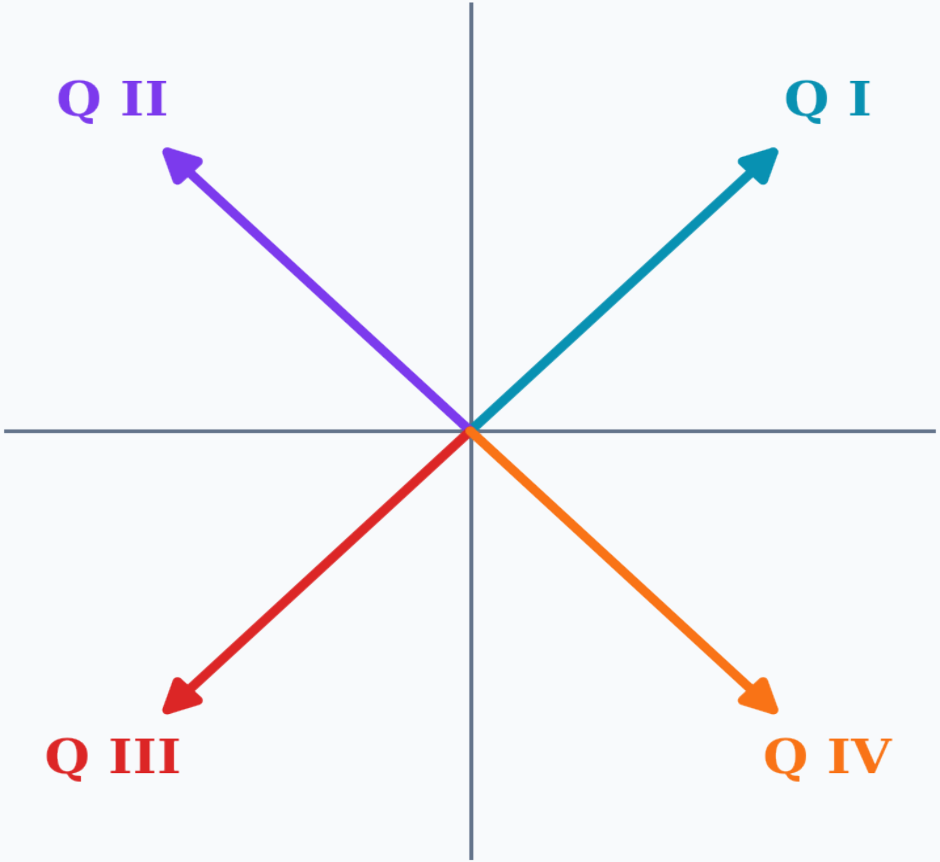
**Example:** $v = \langle -3, 4 \rangle$

$|v| = \sqrt{(9 + 16)} = \sqrt{25} = 5$

$\tan^{-1}(4/-3) \approx -53.1°$
Q II ($a < 0$, $b > 0$): add 180°
$\theta \approx -53.1° + 180° = 126.9°$



# Direction Angle — Quadrant Adjustment




## One Rule for $\theta$

Compute $\alpha = \tan^{-1}(b/a)$, then:

**if $a < 0 \;\rightarrow\; \theta = \alpha + 180°$**

(Q II or Q III)

**else if $b < 0 \;\rightarrow\; \theta = \alpha + 360°$**

(Q IV)

**otherwise $\rightarrow\; \theta = \alpha$**

(Q I)

# Direction Angle of ai + bj

Find the direction angle of $v = -5i + 12j$.

**Convert to components:**

$v = -5i + 12j \rightarrow \langle -5, 12 \rangle \quad (a = -5, b = 12)$

**Find magnitude:**

$|v| = \sqrt{(25 + 144)} = \sqrt{169} = 13$

**Find direction:**

$\tan^{-1}(12/-5) \approx -67.4°$

$a < 0, b > 0 \rightarrow$ Q II: add 180°

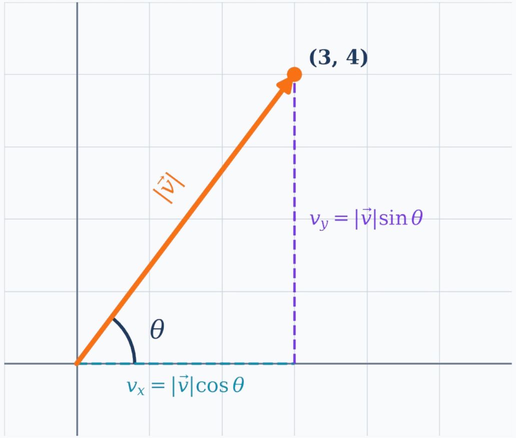
$\boldsymbol{\theta \approx -67.4° + 180° = 112.6°}$

# Components from |v| and θ

## Given |v| and θ

- $v_x = |v| \cos\theta$

$v_y = |v| \sin\theta$
$v = \langle |v| \cos\theta, |v| \sin\theta \rangle$

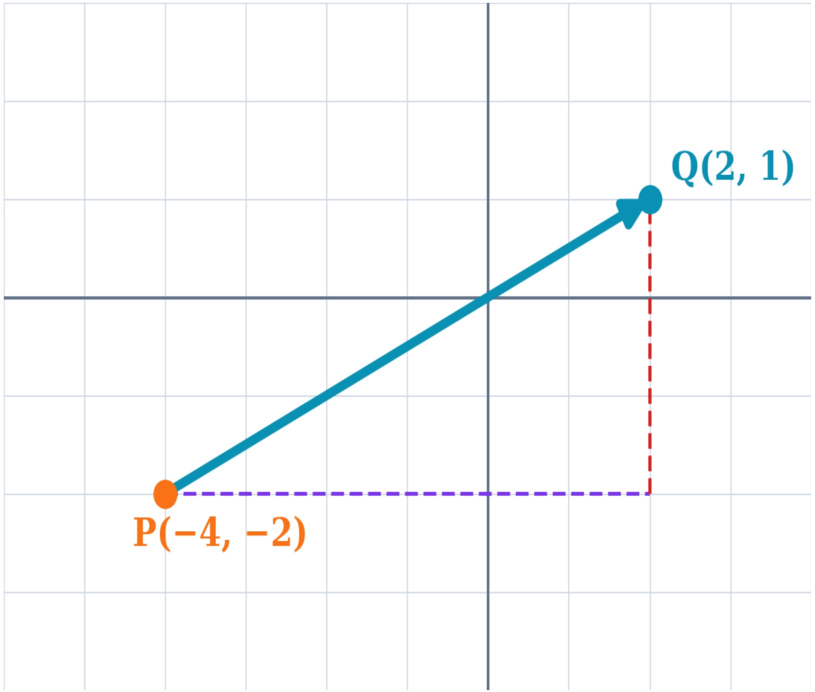



**Example: |v| = 5, θ = 53.1°**

$v_x = 5 \cos 53.1° \approx 3$
$v_y = 5 \sin 53.1° \approx 4$
$v = \langle 3, 4 \rangle$

# You Try

Vector v goes from P(−4, −2) to Q(2, 1).
Find:
(a) component form (b) magnitude
(c) direction angle




(a) $v = \langle 2 - (-4), 1 - (-2) \rangle = \langle 6, 3 \rangle$

(b) $|v| = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5} \approx 6.71$

(c) $\theta = \tan^{-1}(3 / 6) \approx 26.57°$ (Q I, no adjustment)

# Test Prep — Vector Operations

**Test 3, Problem 9:** Vector addition, scalar multiplication, magnitude, and direction angle calculations.

## Part A: Component form

Given: $P(3, -1)$ to $Q(-2, 4)$

$v = \langle -2-3, 4-(-1) \rangle$

$\mathbf{v = \langle -5, 5 \rangle}$

$|v| = \sqrt{(25+25)} = \sqrt{50} = 5\sqrt{2}$

$\theta = \tan^{-1}(5/-5)+180° = -45°+180°$

$\boldsymbol{\theta = 135°}$

## Part B: Operations

Given: $u = \langle 2, -3 \rangle$, $v = \langle -1, 4 \rangle$

$\mathbf{u + v = \langle 1, 1 \rangle}$

$2u - 3v =$

$\mathbf{\langle 4,-6 \rangle - \langle -3,12 \rangle = \langle 7, -18 \rangle}$

$|2u - 3v| = \sqrt{(49+324)} = \sqrt{373}$

$\mathbf{\approx 19.3}$

# Module 13 Summary

5 ALEKS Topics | Section 8.4

## Components

$v = \langle x_2 - x_1, y_2 - y_1 \rangle$ from points, or $v = \langle |v|\cos\theta, |v|\sin\theta \rangle$ from magnitude/direction

## Operations

Add/subtract component-wise, scalar multiplication multiplies each component

## Magnitude & Direction

$|v| = \sqrt{a^2+b^2}$, $\theta = \tan^{-1}(b/a)$ adjusted for quadrant (Q II/III: +180°, Q IV: +360°)

## Key for Test 3

Problem 9: component form, vector operations ($2u - 3v$), magnitude, and direction angle with quadrant adjustment

**Next: Unit 3 Review → Test 3**