Module 12

Section 7.1

Applications of Right Triangle Trigonometry

Section 7.1

Applications and Models

7 ALEKS Skills

Section 7.1 — Skills Overview

1. Using a trig ratio to find a side length
2. Word problem with one right triangle
3. Trig functions and d = rt in real-world problems
4. Using a trig ratio to find an angle measure
5. Angles of elevation or depression
6. Solving a right triangle completely
7. Word problem with two right triangles

Note: Skills 1 & 2 also appeared in Module 10 (Section 5.2) — counted once toward your ALEKS total.

Problem-Solving Strategy

1. Read carefully — identify what is given and what is asked
2. Draw a diagram — label all known sides and angles
3. Identify the right triangle(s) in the problem
4. Choose the trig ratio: SOH CAH TOA
5. Set up the equation and solve for the unknown
6. Check — does the answer make sense? Include units.

Finding a Side Length with Trig

Hypotenuse = 20 m, angle = 42°. Find the opposite side.

Step 1: Identify

Know: hypotenuse = 20, angle = 42°. Need: opposite side → use sin (SOH)

Step 2: Set up

sin 42° = opposite / 20

Step 3: Solve

opposite = 20 × sin 42° = 20 × 0.6691 ≈ 13.38 m

Check: 13.38 < 20 ✓ (opposite < hypotenuse)

Finding an Angle Using Inverse Trig

Opposite = 7, hypotenuse = 15. Find angle A.

sin A = 7/15 = 0.4667
A = sin⁻¹(0.4667) ≈ 27.8°

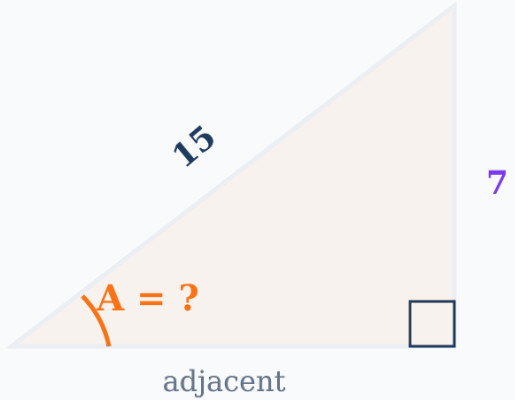
sin A = opp / hyp = 7 / 15 ≈ 0.4667

A = $\sin^{-1}(0.4667)$

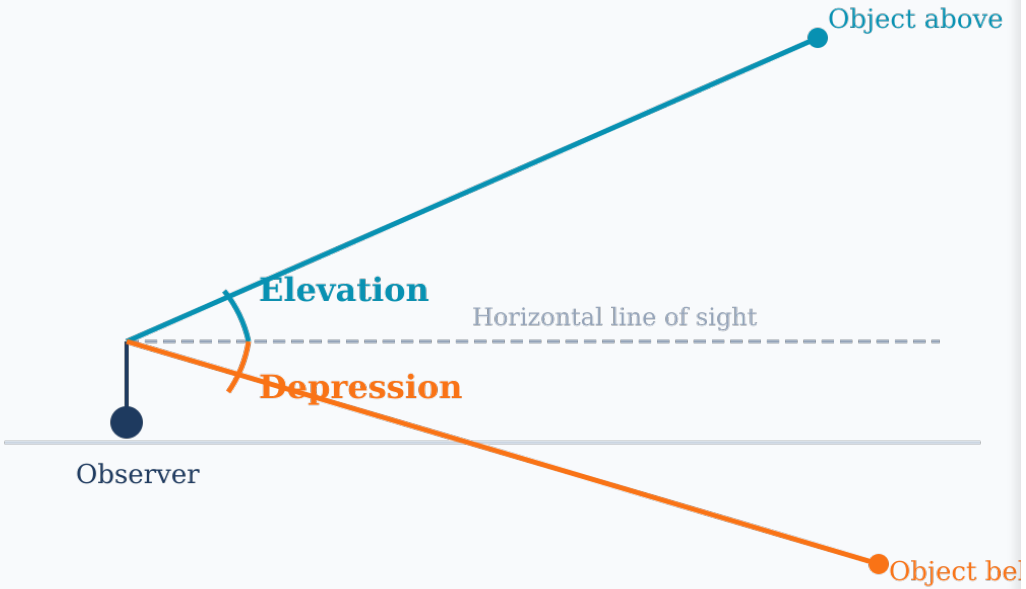
A ≈ 27.8°

Use $\sin^{-1}$ (arcsin), $\cos^{-1}$, or $\tan^{-1}$ to find angles.

Make sure calculator is in DEGREE mode!



Angles of Elevation and Depression



Key Facts

Elevation: looking UP from horizontal

Depression: looking DOWN from horizontal

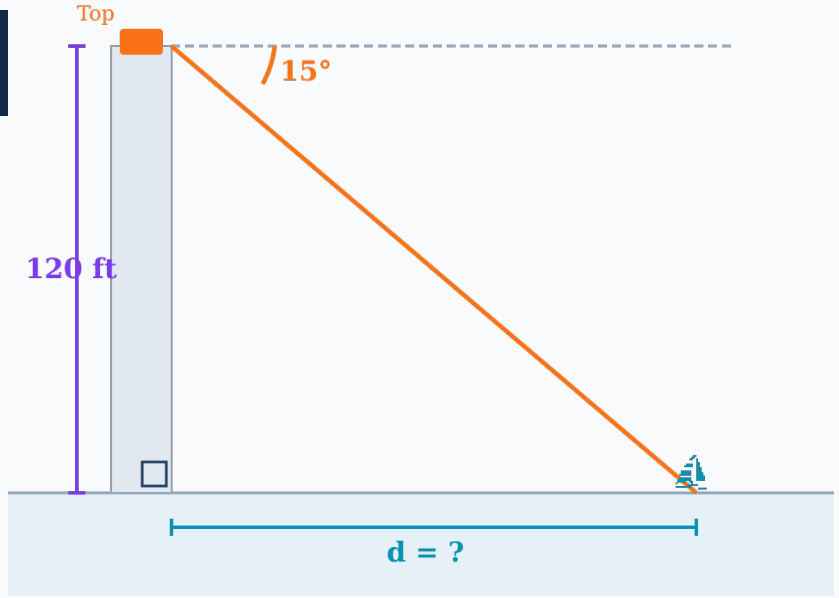
Both measured from the horizontal (never from vertical)

Angle of elevation = angle of depression (alternate interior angles)

Always draw the horizontal at the observer's eye level.

Example — Angle of Depression

From a 120 ft lighthouse top, depression to a ship is 15°. Find the distance.



Depression from top = elevation from ship = 15°

Set up:

tan 15° = opposite / adjacent = 120 / d

Solve:

d = 120 / tan 15°

d = 120 / 0.2679

d ≈ 447.9 ft

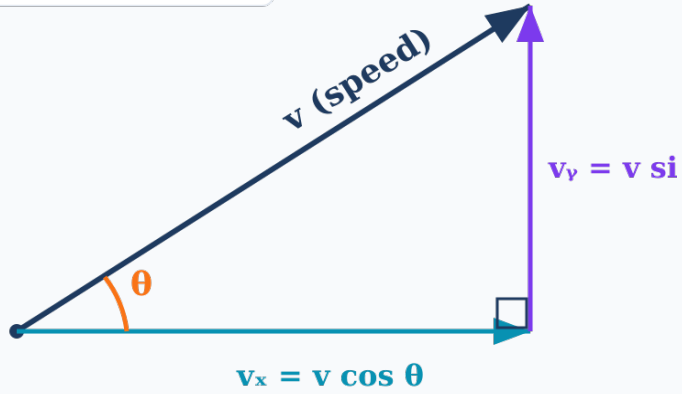
Trig Functions and d = rt

d = rt combined with trig lets us resolve motion into horizontal and vertical components.

d = r × t

$d_x = v \cos\theta \times t$

$d_y = v \sin\theta \times t$



Horizontal

$$d_x = v \cos\theta \times t$$

Vertical

$$d_y = v \sin\theta \times t$$

Example — Trig and d = rt

A plane flies at 250 mph, 30° north of east, for 2 hours. How far N and E?

Total distance:

d = rt = 250 × 2 = 500 miles

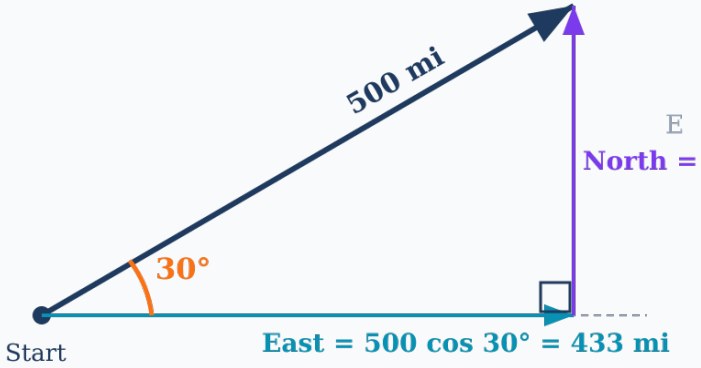
East (horizontal):

$d_x = 500 \cos 30° = 500 \times 0.866 \approx 433$ mi

North (vertical):

$d_y = 500 \sin 30° = 500 \times 0.5 = 250$ mi

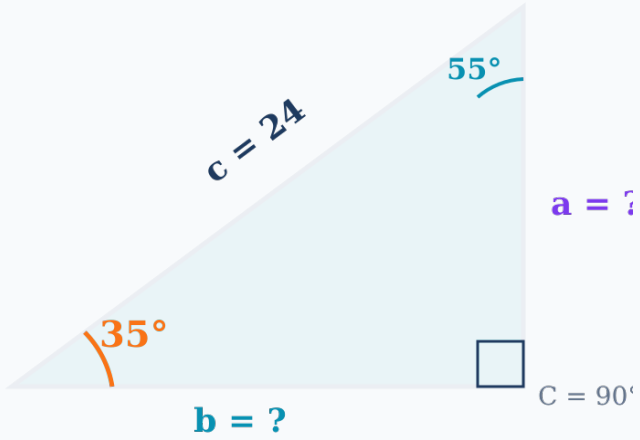
Check: $\sqrt{(433^2 + 250^2)} = \sqrt{250{,}000} = 500$ ✓



Solving a Right Triangle

"Solve" a triangle: Find all 3 sides and all 3 angles.
You need at least one side plus one other known part.

1. Third angle: A + B = 90° → B = 90° − A
2. Use trig ratios to find unknown sides
3. Verify with $a^2 + b^2 = c^2$



Example — Solving a Right Triangle

Given: A = 35°, c = 24 (hypotenuse), C = 90°. Find B, a, and b.

Find B:

$$B = 90^\circ - 35^\circ = 55^\circ$$

Find a (opposite A):

$a = c \sin A = 24 \times \sin 35^\circ = 24 \times 0.5736 \approx 13.77$

Find b (adjacent to A):

$b = c \cos A = 24 \times \cos 35^\circ = 24 \times 0.8192 \approx 19.66$

Check: $13.77^2 + 19.66^2 = 189.6 + 386.5 = 576.1 \approx 24^2 = 576$ ✓

Solution: A = 35°, B = 55°, C = 90°, a ≈ 13.77, b ≈ 19.66, c = 24

Word Problem — Two Right Triangles

Elevation to a building top is 31° from one point, 53° from 50 ft closer. Find the height.

Set up two equations:

Closer: tan 53° = h / d

Farther: tan 31° = h / (d + 50)

Solve: h = d tan 53° = (d+50) tan 31°

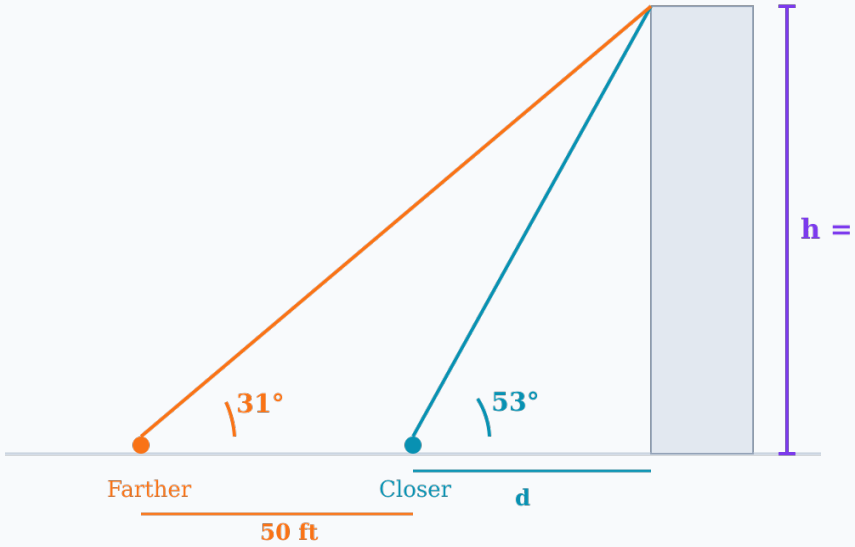
d(1.327) = d(0.601) + 50(0.601)

d(0.726) = 30.04

d ≈ 41.4 ft

h = 41.4 × tan 53° ≈ 41.4 × 1.327

h ≈ 54.9 ft



Test Prep — Solving Triangles (Exact Values)

Test 3, Problem 5: Use special triangles to find exact side lengths (not decimal approximations).

Example: 30-60-90 with adj = 5, θ = 60°

tan 60° = opp / 5

opp = 5 tan 60° = 5√3

cos 60° = 5 / hyp

hyp = 5 / cos 60° = 5 / (1/2) = 10

Key: Use exact trig values

tan 60° = √3, cos 60° = 1/2

Example: 45-45-90 with hyp = 14

sin 45° = opp / 14

opp = 14 sin 45° = 14 × √2/2 = 7√2

Both legs equal in 45-45-90:

adj = opp = 7√2

Key: Use exact trig values

sin 45° = cos 45° = √2/2

Test Prep — Solve a Right Triangle

Test 3, Problem 6: Given two sides, find all angles and the third side. Example: a = 11, c = 35.

Step 1: Find angle A

$\sin A = a/c = 11/35 \approx 0.3143$

$$A = \sin^{-1}(0.3143) \approx 18.3^\circ$$

Step 2: Find angle B

$$B = 90^\circ - 18.3^\circ = 71.7^\circ$$

Step 3: Find side b

$b = \sqrt{(c^2 - a^2)} = \sqrt{(1225 - 121)} = \sqrt{1104} \approx 33.2$

Solution: A ≈ 18.3°, B ≈ 71.7°, C = 90°, a = 11, b ≈ 33.2, c = 35

Test Prep — Elevation & Depression

Test 3, Problem 8: From two points, angles of elevation to a tower give two right triangles with a shared height.

General approach for two-triangle elevation problems:

1. Let h = unknown height, d = closer distance
2. **Closer: tan(large angle) = h / d**
3. **Farther: tan(small angle) = h / (d + gap)**
4. Both equal h, so set right sides equal:

   $d \times \tan(\text{large}) = (d + \text{gap}) \times \tan(\text{small})$
5. Solve for d, then h = d × tan(large angle)

This pattern appears on the test — practice setting up the two equations!

Module 12 Summary

7 ALEKS Topics | Section 7.1

Core Skills

Find sides (SOH CAH TOA), find angles (inverse trig), solve complete right triangles

Applications

Elevation/depression word problems, d = rt with trig components, two-triangle problems

Key for Test 3

Problem 5 (exact values with special triangles), Problem 6 (solve right triangle from two sides), Problem 8 (two-triangle elevation)

Next: Module 13 — Section 8.4 (Vectors in Component Form)