Module 10

Sections 5.1 & 5.2

Angles and Right Triangle Trigonometry

Section 5.1

Angles and Their Measure

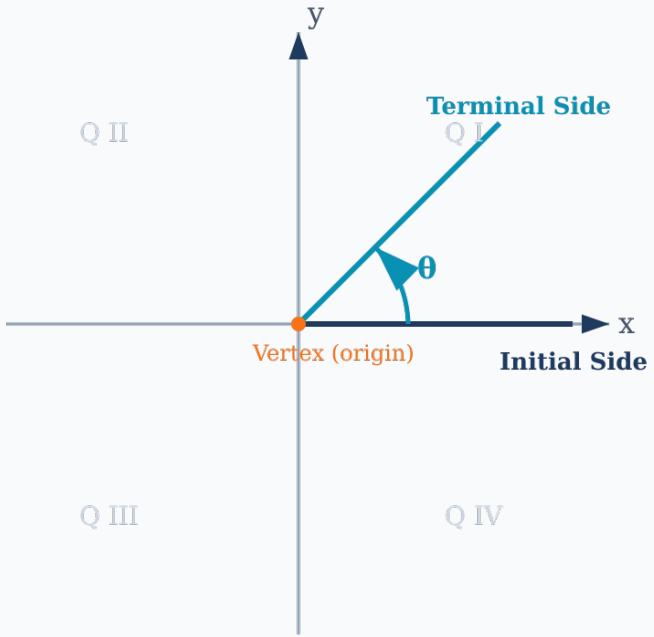
5 ALEKS Skills

Section 5.1 — Skills Overview

1. Sketching angles with $|\theta| < 360°$ in standard position
2. Sketching angles in degrees & finding coterminal angles
3. Sketching angles in radians & finding coterminal angles
4. Sketching angles with $|\theta| > 360°$ in standard position
5. Arc length and central angle measure

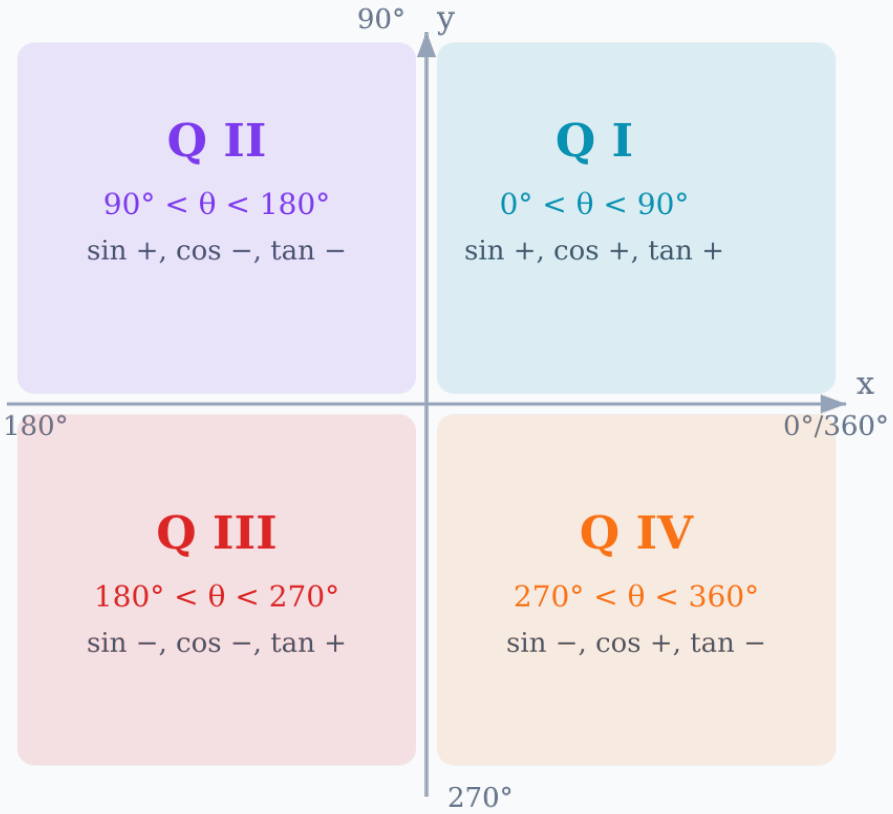
Angles in Standard Position

Standard Position: An angle with its vertex at the origin and its initial side along the positive x-axis.



- Positive angles → counterclockwise rotation
- Negative angles → clockwise rotation
- Terminal side: the ray after rotation
- Initial side: always on the positive x-axis

Quadrants and Angle Ranges



Q I: 0° – 90°

All trig values positive

Q II: 90° – 180°

Only sin positive

Q III: 180° – 270°

Only tan positive

Q IV: 270° – 360°

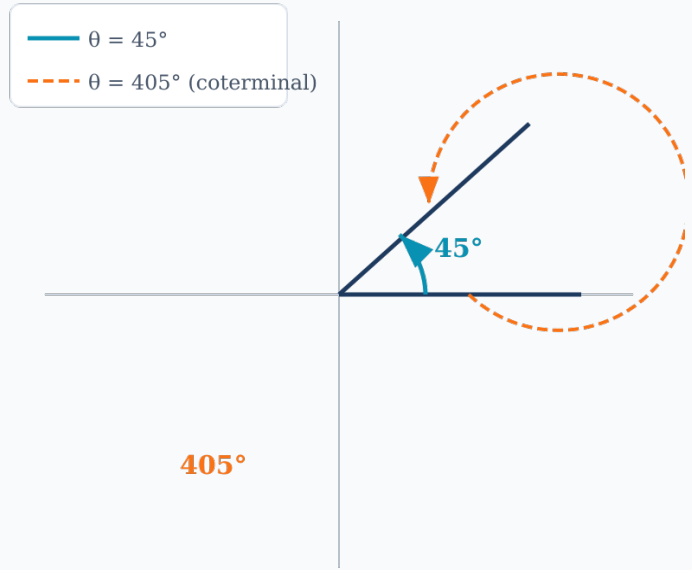
Only cos positive

Mnemonic: All Students Take Calculus

Coterminal Angles — Degrees

Coterminal Angles: Angles that share the same terminal side. Found by adding/subtracting 360°.

Formula: $\theta \pm 360° \cdot n$ (n = any positive integer)



Example 1: 45° + 360° = 405° (coterminal)

Example 2: −30° + 360° = 330° (coterminal)

Example 3: −97° + 360° = 263° (positive coterminal)

Every angle has infinitely many coterminal angles.

Coterminal Angles — Radians

Formula: $\theta \pm 2\pi n$ (n = any positive integer)

Examples

- $\pi/4 + 2\pi = 9\pi/4$ (coterminal)
- $-\pi/3 + 2\pi = 5\pi/3$ (coterminal)
- $9\pi/7 - 2\pi = 9\pi/7 - 14\pi/7 = -5\pi/7$

Key Conversions

- 360° = 2π radians
- 180° = π radians
- **Degrees → Radians: deg × (π / 180)**
- **Radians → Degrees: rad × (180 / π)**

Test Prep: Be comfortable finding coterminal angles for radians like 9π/7 — this appears on Test 3, Problem 2b.

Sketching Angles with $|\theta| > 360°$

Strategy: Subtract (or add) 360° until the angle is between 0° and 360°, then sketch that coterminal angle.

Example: $\theta = 495°$

$495° - 360° = 135°$

One full rotation + 135°

Terminal side is in Quadrant II

(same position as 135°)

Example: $\theta = -750°$

$-750° + 2(360°) = -750° + 720°$

$= -30° \rightarrow$ add $360° \rightarrow 330°$

Terminal side is in Quadrant IV

(same position as 330°)

Arc Length and Central Angle

Arc Length

$$s = r\theta$$

θ must be in radians

Central Angle

$$\theta = s / r$$

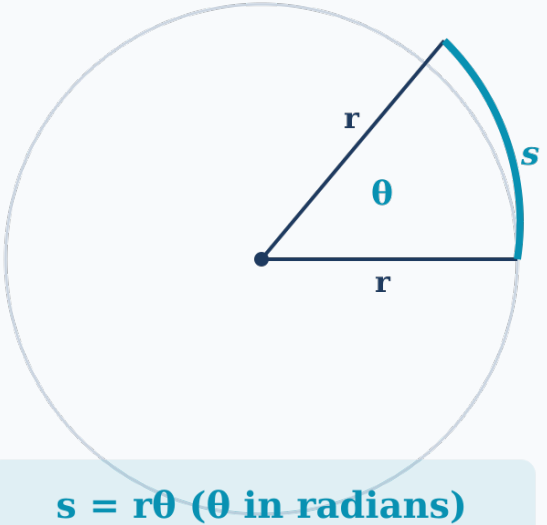
Result is in radians

Conversion

$$\theta(\text{rad}) = \theta(\text{deg}) \times \pi / 180$$

Always convert first!

Full circle: $s = 2\pi r$ (when $\theta = 2\pi$)



$s = r\theta$ (θ in radians)

Example — Arc Length

A circle has radius 12 cm. Find the arc length subtended by a central angle of 150°.

1

Step 1: Convert to radians

$\theta = 150° \times (\pi / 180) = 5\pi/6 \approx 2.618$ rad

2

Step 2: Apply formula

$s = r\theta = 12 \times 5\pi/6$

3

Step 3: Calculate

$s = 10\pi \approx 31.4$ cm

Section 5.2

Right Triangle Trigonometry

8 ALEKS Skills

Section 5.2 — Skills Overview

1. Using a calculator to approximate sin, cos, and tan values
2. Trig ratios: Numbers for side lengths
3. Trig ratios: Variables for side lengths
4. Pythagorean Theorem to find a trig ratio
5. Pythagorean Theorem for several trig ratios
6. Using a trig ratio to find a side length
7. Word problem with one right triangle
8. Word problem with two right triangles

Right Triangle Trigonometry

sin A = Opposite / Hypotenuse

SOH

cos A = Adjacent / Hypotenuse

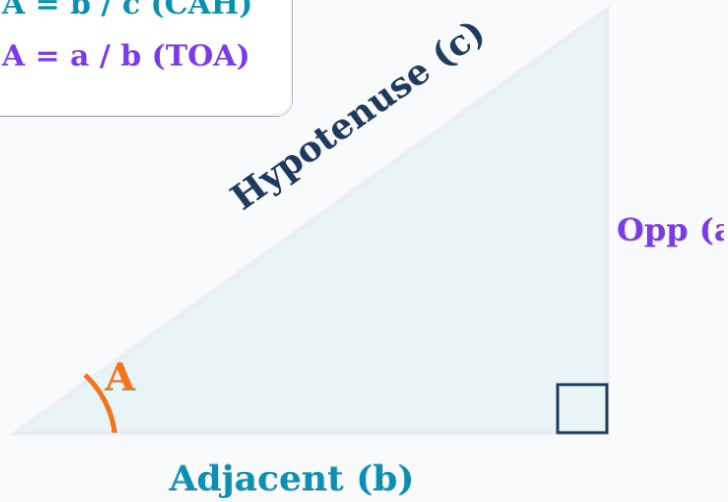
CAH

tan A = Opposite / Adjacent

TOA

These ratios are identical for all similar right triangles sharing angle A.

sin A = a / c (SOH)
cos A = b / c (CAH)
tan A = a / b (TOA)



Reciprocal Trigonometric Functions

csc A = 1 / sin A = Hypotenuse / Opposite

Quotient: tan A = sin A / cos A

sec A = 1 / cos A = Hypotenuse / Adjacent

cot A = 1 / tan A = Adjacent / Opposite

Quotient: cot A = cos A / sin A

These six functions — sin, cos, tan, csc, sec, cot — form the foundation of trigonometry.

Using a Calculator for Trig Values

- Check your mode — set to DEGREE or RADIAN as needed
- Use the sin, cos, tan buttons directly
- For csc, sec, cot: compute the reciprocal
- Round to 4 decimal places unless instructed otherwise

Example: $\theta = 37^\circ$

$\sin 37^\circ \approx 0.6018$

$\cos 37^\circ \approx 0.7986$

$\tan 37^\circ \approx 0.7536$

$\csc 37^\circ = 1/0.6018 \approx 1.6616$

$\sec 37^\circ = 1/0.7986 \approx 1.2521$

$\cot 37^\circ = 1/0.7536 \approx 1.3270$

Example — Trig Ratios (Numbers)

Right triangle with legs 5 and 12, hypotenuse 13. Find all six trig ratios for angle A (opposite side 5).

sin A = 5/13
≈ 0.3846

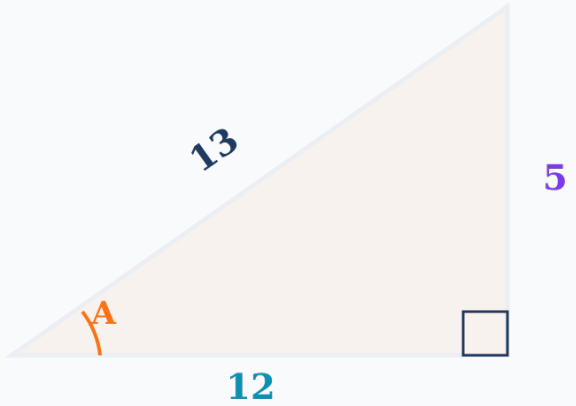
csc A = 13/5
= 2.6

cos A = 12/13
≈ 0.9231

sec A = 13/12
≈ 1.0833

tan A = 5/12
≈ 0.4167

cot A = 12/5
= 2.4



Check: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$ ✓

Using Pythagorean Theorem for Trig Ratios

Given one leg = 8 and hypotenuse = 17, find sin A, cos A, and tan A for angle A opposite the unknown leg.

Step 1: Find the missing side

$a^2 + b^2 = c^2 \rightarrow b^2 = 17^2 - 8^2 = 289 - 64 = 225 \rightarrow b = 15$

Step 2: Write the ratios

$\sin A = 15/17 \approx 0.8824$ (opposite / hypotenuse)

$\cos A = 8/17 \approx 0.4706$ (adjacent / hypotenuse)

$\tan A = 15/8 = 1.875$ (opposite / adjacent)

Test Prep — Finding sin θ and tan θ

Test 3, Problem 1: Given cos θ = 9/41, find sin θ and tan θ.

Method: Right Triangle

cos θ = adj/hyp = 9/41

Find opposite side:

$opp^2 = 41^2 - 9^2 = 1681 - 81 = 1600$

opp = √1600 = 40

Answers:

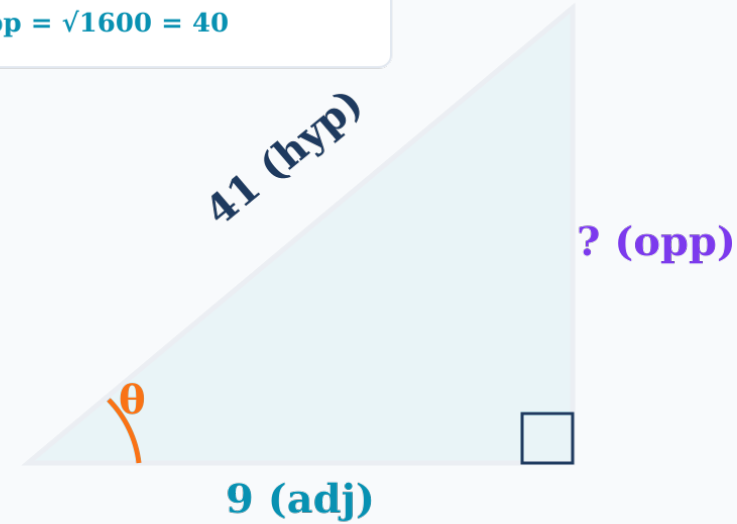
sin θ = 40/41

tan θ = 40/9

cos θ = 9/41 = adj/hyp

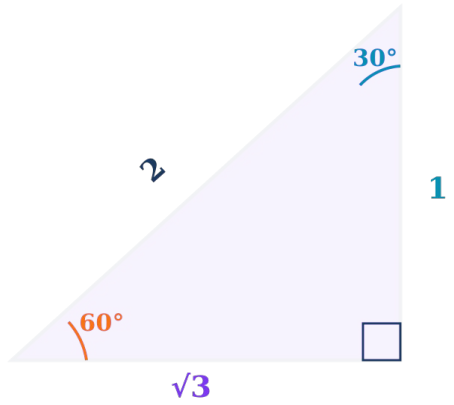
opp = √(41² − 9²) = √(1681 − 81)

opp = √1600 = 40



Special Right Triangles

30-60-90 Triangle

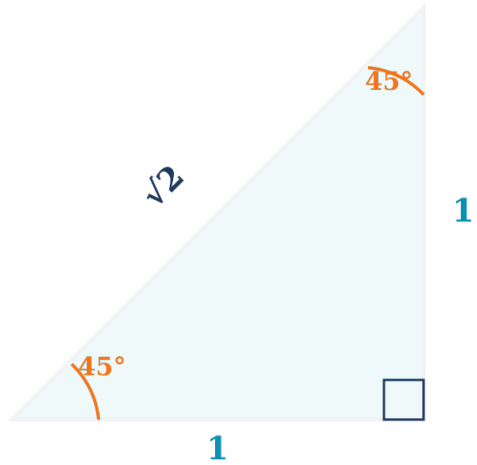


Ratio: 1 : √3 : 2

sin 30° = 1/2, cos 30° = √3/2

sin 60° = √3/2, cos 60° = 1/2

45-45-90 Triangle



Ratio: 1 : 1 : √2

sin 45° = cos 45° = √2/2

tan 45° = 1

Finding a Side Length Using Trig

A right triangle has hypotenuse 20 m and one angle of 35°. Find the side opposite the 35° angle.

Identify what you know and what you need:

Know: hypotenuse = 20 m, angle = 35°

Need: opposite side → Use sin (SOH)

Set up and solve:

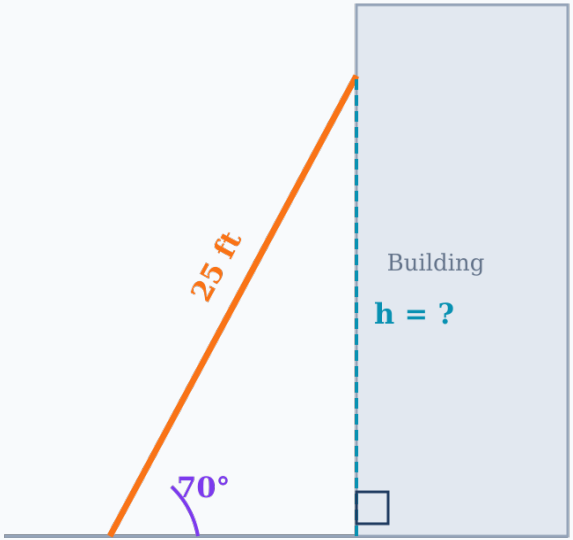
sin 35° = opposite / 20

$$\text{opposite} = 20 \times \sin 35^\circ = 20 \times 0.5736 \approx 11.5 \text{ m}$$

Tip: Always check — the opposite side should be shorter than the hypotenuse.

Word Problem — One Right Triangle

A 25 ft ladder leans against a building at 70° with the ground. How high does it reach?



Draw the triangle:

Ladder = hypotenuse = 25 ft, base angle = 70°

Height = side opposite the 70° angle

Solve:

$\sin 70° = h / 25$

$h = 25 \times \sin 70° = 25 \times 0.9397$

$h \approx 23.5$ ft

Word Problem — Two Right Triangles

From a 150 ft cliff, angle of depression to one boat is 20° and to a second boat (farther) is 12°. How far apart are the boats?

Near Boat (20°)

$\tan 20° = 150 / d_1$

$d_1 = 150 / \tan 20°$

$d_1 \approx 412.1$ ft

Far Boat (12°)

$\tan 12° = 150 / d_2$

$d_2 = 150 / \tan 12°$

$d_2 \approx 705.5$ ft

Distance apart = $d_2 - d_1 = 705.5 - 412.1 \approx$ **293.4 ft**

Trigonometric Identities (Section 5.2)

Reciprocal	$\csc\theta = 1/\sin\theta$	$\sec\theta = 1/\cos\theta$	$\cot\theta = 1/\tan\theta$
Quotient	$\tan\theta = \sin\theta / \cos\theta$	$\cot\theta = \cos\theta / \sin\theta$	
Pythagorean	$\sin^2\theta + \cos^2\theta = 1$		
Pythagorean (derived)	$1 + \tan^2\theta = \sec^2\theta$	$1 + \cot^2\theta = \csc^2\theta$	
Co-function	$\sin(90°-\theta) = \cos\theta$	$\cos(90°-\theta) = \sin\theta$	$\tan(90°-\theta) = \cot\theta$

Test 3 reference sheet provides: **$\sin^2x + \cos^2x = 1$, $1 + \tan^2x = \sec^2x$, Area = ½bc sin A**

Finding All Trig Ratios from One

Given $\sin\theta = 3/5$, find $\cos\theta$, $\tan\theta$, $\csc\theta$, $\sec\theta$, and $\cot\theta$.

$\sin\theta = \text{opp}/\text{hyp} = 3/5$

So: opp = 3, hyp = 5

Find adj:

$\text{adj} = \sqrt{(5^2 - 3^2)} = \sqrt{(25 - 9)} = \sqrt{16} = 4$

All Six Ratios:

$\cos\theta = 4/5 = 0.8$

$\tan\theta = 3/4 = 0.75$

$\csc\theta = 5/3 \approx 1.667$

$\sec\theta = 5/4 = 1.25$

$\cot\theta = 4/3 \approx 1.333$

Test Prep — Exact Lengths from Special Triangles

Test 3, Problem 5: Find exact values using 30-60-90 and 45-45-90 relationships.

30-60-90 Example

Given: vertical side = 17, angle = 30°
Find y (horizontal) and r (hypotenuse)

Side 17 is opposite 30° (short leg)

Hypotenuse = 2 × short leg
$r = 2 \times 17 = 34$

Long leg = short leg × √3
$y = 17\sqrt{3}$

45-45-90 Example

Given: hypotenuse = 24, angle = 45°
Find x and y (both legs)

In a 45-45-90, both legs are equal

hyp = leg × √2
24 = leg × √2
$\text{leg} = 24/\sqrt{2} = 24\sqrt{2}/2 = 12\sqrt{2}$

$x = y = 12\sqrt{2}$

Module 10 Summary

13 ALEKS Topics | Sections 5.1 & 5.2

Section 5.1

Angles in standard position, coterminal angles (degrees & radians), arc length $s = r\theta$

Section 5.2

SOH-CAH-TOA, trig ratios, calculator usage, Pythagorean Theorem with trig, special triangles (30-60-90, 45-45-90), word problems

Key for Test 3

Finding sin/tan given $\cos\theta = 9/41$ (Problem 1), coterminal & reference angles (Problem 2), exact values with special triangles (Problem 5)

Next: Module 11 — Section 5.3 (Reference Angles & Trig Values by Quadrant)