Test 2 Review

Sections 4.2, 4.3 & Geometry

Exponentials, Logs, Angles, Areas, Circles & Speed

Test 2 — Topics at a Glance

1. Compound Interest (discrete & continuous)
2. Exponential ↔ Logarithmic form conversions
3. Solving exponential & log equations
4. Angle conversions: DMS ↔ decimal degrees, radians ↔ degrees
5. Arc length ($s = r\theta$) and sector area ($A = \frac{1}{2}r^2\theta$)
6. Triangle area: $\frac{1}{2}bh$ and Heron's formula
7. Trapezoid area: $\frac{1}{2}(b_1+b_2)h$ (with Pythagorean Thm for height)
8. Parallel lines & proportional segments
9. Circle theorems: inscribed angles, central angles, intersecting chords
10. Linear & angular speed: $v = r\omega$

Compound Interest

Discrete

A = P(1 + r/n)^(nt)

P = principal, r = rate (decimal), n = compounds/yr, t = years

Continuous

A = Pe^(rt)

Same P, r, t. e ≈ 2.71828. Use when "compounded continuously."

Example: $2,500 at 3% for 20 years

Monthly (n=12): A = 2500(1 + 0.03/12)^(240) = 2500(1.0025)^240 ≈ $4,551.89

Continuous: A = 2500·e^(0.6) = 2500 × 1.82212 ≈ $4,555.30

Calculator tip: Use parentheses! Enter (1.0025)^240, not 1.0025^240.

Exponential ↔ Logarithmic Form

b^y = x ⇔ log_b(x) = y The base stays the base!

Exp → Log

25^(5/2) = 3125 → $\log_{25}(3125) = 5/2$

Log → Exp

$\log_4(2048) = 5.5$ → 4^5.5 = 2048

Solve: ln x = 4

$x = e^4 \approx 54.598$

Solve: 2^x = 13

$x = \ln(13)/\ln(2) \approx 3.700$

Angle Conversions

DMS ↔ Decimal

DMS → Dec: D + M/60 + S/3600

37°41'11" = 37 + 41/60 + 11/3600 = 37.6864°

Dec → DMS: take decimal × 60 each step

36.3492° → 36° + 0.3492×60 = 20.952'

→ 20' + 0.952×60 = 57" → 36°20'57"

Radians ↔ Degrees

Deg → Rad: × π/180

163.9° × π/180 ≈ 2.86 rad

Rad → Deg: × 180/π

2.7 rad × 180/π ≈ 154.7°

Common: 30°=π/6 45°=π/4 60°=π/3 90°=π/2 180°=π 360°=2π

Arc Length & Sector Area

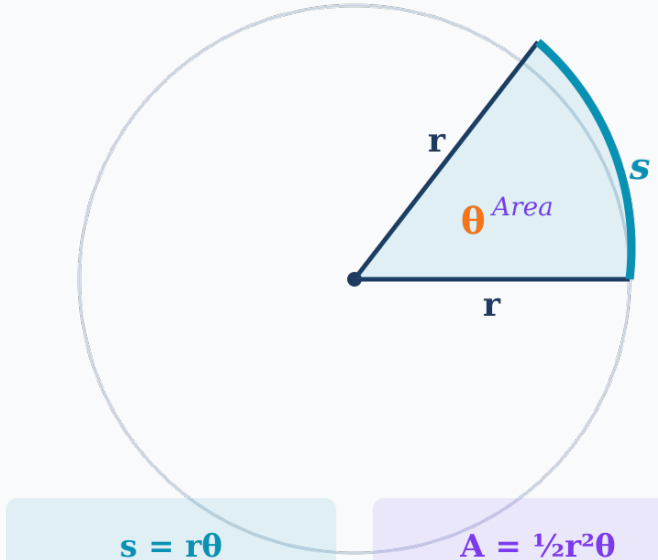
Arc Length $s = r\theta$

r=5.7, θ=65° → θ=1.1345 rad → s=5.7×1.1345 ≈ 6.5

Sector Area $A = \frac{1}{2}r^2\theta$

r=10, θ=3π/5 → A=½×100×(3π/5) ≈ 94.2

⚠ θ MUST be in radians! Convert degrees first: θ_rad = θ_deg × π/180



$s = r\theta$

$A = \frac{1}{2}r^2\theta$

Triangle Area: Heron's Formula

$$s = (a+b+c)/2 \qquad A = \sqrt{[s(s-a)(s-b)(s-c)]}$$

Example: a = 1.5 cm, b = 1.3 cm, c = 2.1 cm

$s = (1.5 + 1.3 + 2.1)/2 = 4.9/2 = 2.45$

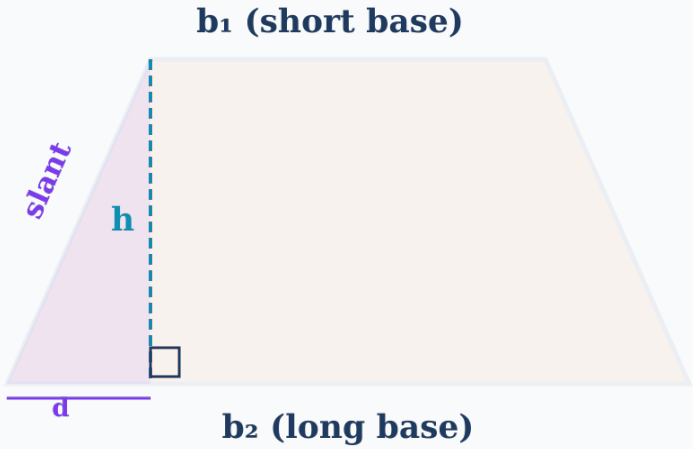
$s-a = 0.95, \quad s-b = 1.15, \quad s-c = 0.35$

$$A = \sqrt{(2.45 \times 0.95 \times 1.15 \times 0.35)} = \sqrt{(0.937)} \approx 1.0 \text{ cm}^2$$

Use Heron's when you know all 3 sides but NOT the height.

Trapezoid Area (Multi-Step)

$A = \frac{1}{2}(b_1 + b_2)h$ (may need Pythagorean Thm for h)



Ex: b_1=4.5, b_2=10.1, slant=7.0 m

Step 1: $d = b_2 - b_1 = 5.6$ m

Step 2: $h^2 + 5.6^2 = 7.0^2$

$h^2 = 49 - 31.36 = 17.64 \rightarrow h = 4.2$ m

Step 3: $A = \frac{1}{2}(4.5+10.1)(4.2)$

$A = \frac{1}{2}(14.6)(4.2) = 30.66$ m²

Parallel Lines & Proportional Segments

If parallel lines cut transversals, corresponding segments are proportional: AB/BD = AC/CE

Example: AB = 40, BD = 45, AC = 37. Find CE and DC.

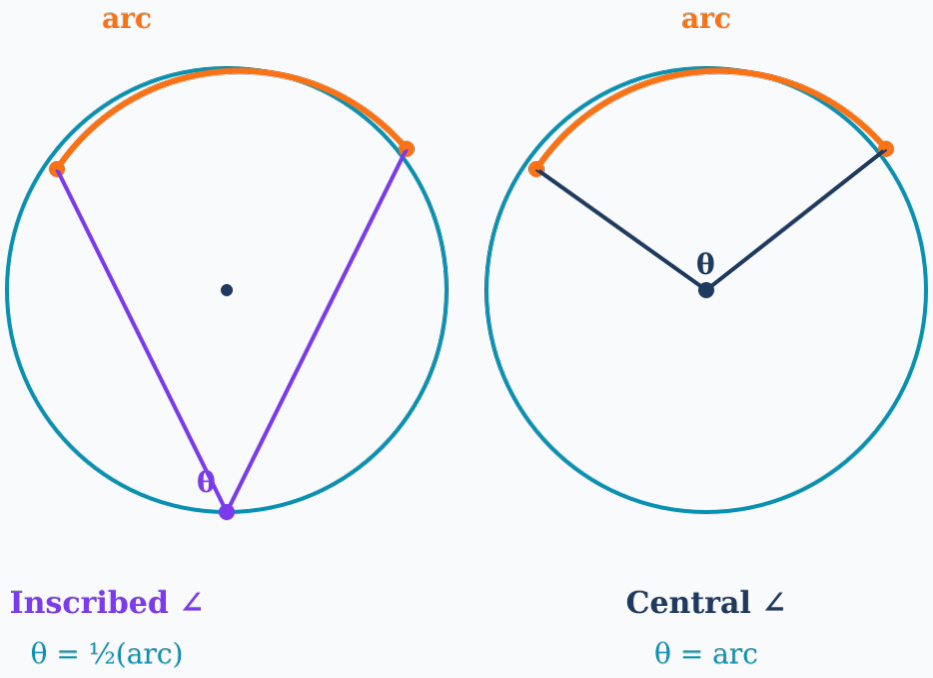
Set up proportion:

40/45 = 37/CE

CE = 37 × 45/40 = 41.6 m

DC = CE – AC = 41.6 – 37 = 4.6 m

Circle Theorems: Arcs & Angles



Central angle = intercepted arc

Inscribed angle = ½(intercepted arc)

Intersecting chords: $\angle = \frac{1}{2}(arc_1 + arc_2)$

Semicircle: diameter → arc = 180°

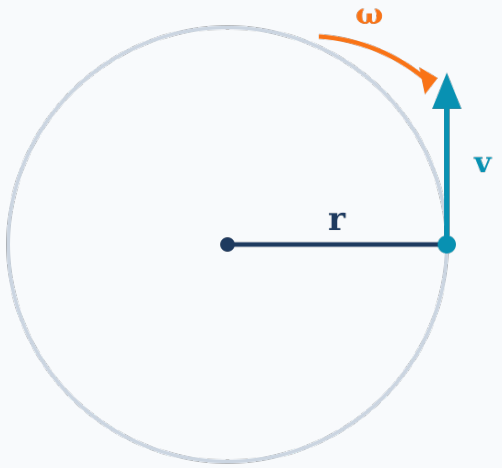
Linear & Angular Speed

Angular speed $\omega = \theta/t$ (rad/time)

1 rev = 2π radians

Linear speed $v = r\omega$

distance per unit time at edge



$\omega = \theta/t$ $v = r\omega$

Example: 350 RPM, r = 3 ft

$\omega = 350 \times 2\pi = 700\pi$ rad/min

$v = 3 \times 700\pi = 2100\pi \approx 6{,}597$ ft/min

$\rightarrow$ **$6{,}597 \times 60 \div 5280 \approx 75.0$ mph**

Practice Quiz — Try Before Checking!

1. $4,000 at 6% for 10 yrs, monthly. Find A. **$7,277.59**

2. Write $\log_3(81)=4$ in exponential form. **$3^4 = 81$**

3. Solve: e^x = 20 (3 dec places). **x = ln20 ≈ 2.996**

4. Convert 45°22'18" to decimal degrees. **45.3717°**

5. Convert 128.75° to DMS. **128°45'00"**

6. Convert 5π/6 to degrees. **150°**

7. Arc length: r=12, θ=40°. **s ≈ 8.4**

8. Sector area: r=7, θ=2π/3. **A ≈ 51.3**

9. Heron's: a=5, b=7, c=9. **A ≈ 17.4**

10. 600 RPM, r=1.5 ft. Find mph. **≈ 64.3 mph**

Formula Cheat Sheet

Interest	$A = P(1+r/n)^{(nt)}$ | $A = Pe^{(rt)}$
Log ↔ Exp	$b^y = x \iff \log_b(x) = y$ | $\ln(a^n) = n \cdot \ln(a)$
DMS	$D + M/60 + S/3600$ | Deg→Rad: $\times \pi/180$ | Rad→Deg: $\times 180/\pi$
Arc/Sector	$s = r\theta$ (rad) | $A = \frac{1}{2}r^2\theta$ (rad) | Always convert θ to radians first!
Triangle	$A = \frac{1}{2}bh$ | Heron's: $s=(a+b+c)/2$, $A=\sqrt{[s(s-a)(s-b)(s-c)]}$
Trapezoid	$A = \frac{1}{2}(b_1+b_2)h$ | May need: $h^2+d^2=\text{slant}^2$, $d=b_2-b_1$
Parallel Lines	Proportional segments: AB/BD = AC/CE
Circles	Central ∠ = arc | Inscribed ∠ = ½(arc) | Chords: $\angle = \frac{1}{2}(\text{arc}_1+\text{arc}_2)$
Speed	$\omega = \theta/t$ | $v = r\omega$ | 1 rev = 2π rad | ft/min → mph: $\times 60 \div 5280$

Good luck on Test 2! Remember: show all work for full credit.