

MATH 130

Geometry: Angles & Polygons

Properties, Measurements, and Formulas for Lines, Angles, and Surfaces

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Learning Goals

By the end of this lesson, you will be able to:

1. Classify and measure angles

Identify acute, obtuse, right, and straight angles; find complements and supplements.

2. Apply parallel line theorems

Use corresponding, alternate interior, and alternate exterior angle relationships to solve for unknowns.

3. Calculate polygon angle sums

Apply interior and exterior angle formulas for convex polygons with n sides.

4. Compute areas of polygons

Find areas of quadrilaterals and triangles using standard formulas including Heron's Formula.

5. Solve similar triangle problems

Use proportional reasoning to find unknown sides and angles in similar polygons; apply to real-world contexts.

6. Convert between degree formats

Convert angles between decimal degrees and degrees-minutes-seconds (DMS) notation used in navigation.

SECTION 1

Foundations: Points, Lines & Angles

Every geometric shape begins with points, lines, and angles — the atomic building blocks of all geometry.

Undefined Terms in Geometry

Euclid attempted to define all geometric terms, but some definitions create challenges. We accept that point, line, and plane are undefined terms — we understand them intuitively.



Point

A location in space with no dimension — no length, width, or height.



Line

A straight path extending infinitely in both directions. Has length but no width.



Plane

A flat surface that extends infinitely in all directions. Has length and width but no depth.

Why “undefined”? Every definition requires prior terms. Point, line, and plane are the starting primitives — we build all of geometry on top of them.

Geometric Notation Conventions

Line AB

A straight path extending in both directions, no endpoints. Notation: $\overleftrightarrow{AB}$ or line through AB.

Ray AB

Starts at endpoint A, passes through B, extends forever. Notation: $\overrightarrow{AB}$. Endpoint listed first.

Segment AB

A finite path connecting two endpoints A and B. Notation: $\overline{AB}$. Has a definite, measurable length.

Angle $\angle ABC$

Two rays meeting at a vertex. Notation: $\angle ABC$ where B is the vertex. Can abbreviate as $\angle B$ if unambiguous. Measured in degrees.

Congruence & Similarity

Congruence $\cong$

Use this notation consistently throughout the course.

Same shape and size. $\triangle ABC \cong \triangle DEF$ means identical in shape and size.

Similarity $\sim$

Same shape, proportional sizes. $\triangle ABC \sim \triangle DEF$: corresponding sides proportional. Order of letters in similarity statement matters!

Rays & Angles

Ray

A half-line — starts at a point (endpoint) and extends infinitely in one direction.

Angle

Two rays sharing a common endpoint (vertex).
The measure is the rotation from one ray to the other.

One complete rotation = 360° | Measured in degrees or radians | Protractor: tool for measuring angles

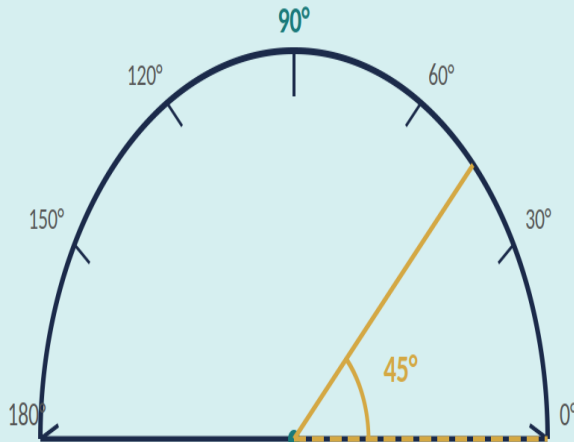
Components of an Angle

Initial Side (R_1): The starting ray of the angle

Terminal Side (R_2): The ending ray after rotation

Vertex: The common endpoint where both rays meet

Measuring Angles



Standard semi-circle protractor

Tools & Units

Angles can be measured in **degrees** or **radians**.

Protractor

A protractor is an instrument used to measure angles (almost always in degree measure).

Key Benchmarks

Full rotation = 360° | Straight angle = 180°

Right angle = 90° | In radians: $360^\circ = 2\pi \text{ rad}$

Types of Angles

Right Angle

Exactly 90°

Straight Angle

Exactly 180°

Acute Angle

Between 0° and 90°

Obtuse Angle

Between 90° and 180°

Supplementary Angles

Two angles whose measures
add up to 180°

Complementary Angles

Two angles whose measures
add up to 90°

Adjacent & Vertical Angles

Adjacent Angles

Two angles that share a common vertex and a common side, but do not overlap.

$\angle AOB$ and $\angle BOC$ are adjacent angles — they share vertex O and ray OB .

Vertical Angles

Two angles formed by intersecting lines that are on opposite sides of the intersection.

Vertical angles are always equal in measure (congruent).

Key Insight: When two lines intersect, they form two pairs of vertical angles. Each pair consists of congruent angles, and any two adjacent angles are supplementary (sum to 180°).

Angle Relationship Map

RELATIONSHIP	DEFINITION & KEY PROPERTY
Vertical	Formed by intersecting lines; opposite each other; always congruent.
Adjacent	Share a vertex and a side; do not overlap; may or may not be supplementary.
Linear Pair	Adjacent + supplementary; sum = 180° ; on a straight line.
Complementary	Any two angles whose measures sum to 90° .
Supplementary	Any two angles whose measures sum to 180° .

Key principle: Vertical angles are always congruent. All other relationships depend on position.

Parallel Lines & Transversals

When a transversal crosses parallel lines, special angle relationships form:

Corresponding Angles

Same position at each intersection.

$$\angle 1 = \angle 5$$

Equal

Alternate Interior Angles

Opposite sides, between the lines.

$$\angle 3 = \angle 6$$

Equal

Alternate Exterior Angles

Opposite sides, outside the lines.

$$\angle 1 = \angle 8$$

Equal

Angles labeled 1–8 at two intersections: t is the transversal, $l \parallel m$ are the parallel lines

Corresponding Segments

When two transversals cross a pair of parallel lines, the segments between those parallel lines are called corresponding segments.

If $l_1 \parallel l_2 \parallel l_3$, then segments a, b are corresponding and c, d are corresponding.

$$a / b = c / d \quad \text{and equivalently} \quad a / c = b / d$$

Example

Assuming the vertical lines are parallel, use proportions to find unknown distances. Worked Example: if $a = 6$, $b = 9$, $c = 8$, find d . Step 1 — Set up: $a/b = c/d \rightarrow 6/9 = 8/d$. Step 2 — Cross-multiply: $6d = 9 \times 8 = 72$. Step 3 — Solve: $d = 12$.

Solving Angle Equations: Method

Method

Step 1 — Identify the relationship

Are the angles equal, supplementary (180°), or related by position?

Step 2 — Write the equation

Corr./alt.int./alt.ext. $\rightarrow$ set equal. Co-interior $\rightarrow$ sum to 180° .

Step 3 — Solve for x , then find the angle

Isolate the variable, then substitute back to get the degree measure.

Step 4 — Check with supplementary if needed

Adjacent angles on a straight line sum to 180° . Use this to find the supplementary angle if asked.

Works for any angle pair from parallel lines:

Corresponding, alt. int., alt. ext. $\rightarrow$ equal

Co-interior $\rightarrow$ supplementary (180°)

Quick Reference

Corr./Alt.int./Alt.ext. $\rightarrow$ set equal

Co-interior $\rightarrow$ sum to 180° , then solve

Worked Example: Angle Equations

Worked Example

Given: $l \parallel m$, cut by transversal t .

$$\angle 2 = (3x + 10)^\circ, \angle 6 = (5x - 20)^\circ$$

Step 1: $\angle 2$ and $\angle 6$ are corresponding $\rightarrow$ equal

Step 2: $3x + 10 = 5x - 20$

Step 3: $30 = 2x \rightarrow x = 15$

$$\angle 2 = 3(15) + 10 = 55^\circ$$

Step 4: Supplementary $\rightarrow \angle 1 = 180^\circ - 55^\circ = 125^\circ$

Example: Parallel Lines & Angles

Problem

Lines l and m are parallel, and t is a transversal.

Given $\angle 2 = 2x + 19^\circ$ and $\angle 7 = 3x + 2^\circ$, find the measure of $\angle 1$.

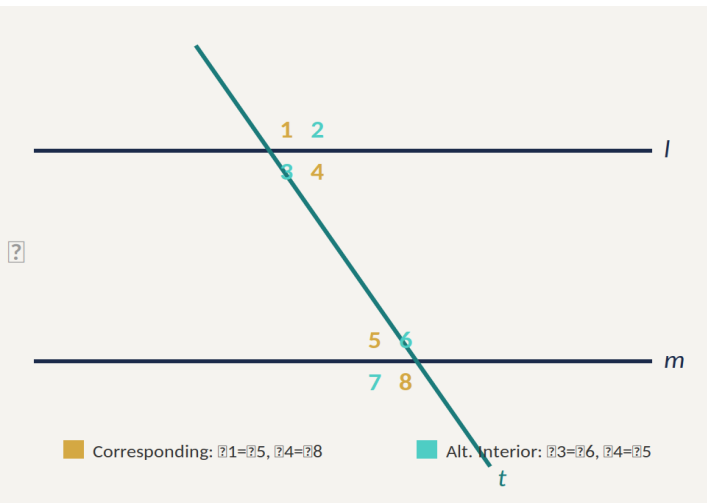
Solution Strategy

Step 1: Identify the relation

Step 2: Set them equal: $2x + 19 = 3x + 2$

Step 3: Find $\angle 2 = 2(17) + 19 = 53^\circ$

Step 4: $\angle 1$ and $\angle 2$ are supplementary



SECTION 2

Polygons & Quadrilaterals

Polygons & quadrilaterals build on angle relationships from Section 1 — now we measure enclosed area and classify shapes.



Polygons

A polygon is a closed region of a plane bounded by three or more straight line segments. Sides must be non-collinear and non-self-intersecting (no crossings). Formulas below apply to convex polygons.

Triangle

3 sides

Quadrilateral

4 sides

Pentagon

5 sides

Hexagon

6 sides

**Heptagon &
beyond**

8+ sides

Polygon Angle Formulas

These formulas apply to convex polygons with n sides.

Interior & Exterior Angles of Polygons

Sum of Interior Angles = $(n - 2) \times 180^\circ$

where n = number of sides

Sum of Exterior Angles = 360° (always, for any convex polygon)

Each Interior Angle of a Regular Polygon = $(n - 2) \times 180^\circ / n$

Interior Angle Sum Formula

Derivation via Triangulation

Any polygon can be divided into triangles by drawing diagonals from one vertex.

A polygon with n sides produces $(n - 2)$ triangles.

Each triangle has an angle sum of 180° .

Sum of Interior Angles = $(n - 2) \times 180^\circ$

Examples

Triangle ($n=3$): $(3-2) \times 180^\circ = 180^\circ$

Quadrilateral ($n=4$): $(4-2) \times 180^\circ = 360^\circ$

Pentagon ($n=5$): $(5-2) \times 180^\circ = 540^\circ$

Hexagon ($n=6$): $(6-2) \times 180^\circ = 720^\circ$

Quick Reference — Interior Angle Sums

Triangle ($n=3$): 180° | Quadrilateral ($n=4$): 360° | Pentagon ($n=5$): 540° | Hexagon ($n=6$): 720°

Heptagon: 900° | Octagon: 1080° | Nonagon: 1260° | Decagon: 1440°

Formula: $(n - 2) \times 180^\circ$ where n = number of sides

Exterior Angle Sum

Key Fact: Sum of Exterior Angles = 360° (always, for any convex polygon)

One exterior angle at each vertex. The sum is always 360° , regardless of the number of sides.

Regular Polygon Interior Angle

Each interior angle of a regular n -gon = $(n - 2) \times 180^\circ / n$

Each exterior angle of a regular n -gon = $360^\circ / n$

Examples

Regular hexagon ($n=6$): interior = 120° , exterior = 60°

Regular octagon ($n=8$): interior = 135° , exterior = 45°

Interior + exterior angle at any vertex = 180° (linear pair)

Regular Polygon Quick Reference

Pentagon (5): interior = 108° , exterior = 72° | Hexagon (6): interior = 120° , exterior = 60°

Octagon (8): interior = 135° , exterior = 45° | Decagon (10): interior = 144° , exterior = 36°

Note: interior + exterior = 180° (always at every vertex – linear pair)

Types of Quadrilaterals

Trapezoid

Exactly two parallel sides (called bases)

Parallelogram

Both pairs of opposite sides are parallel

Rhombus

Parallelogram with four equal sides

Rectangle

Parallelogram with a right angle (all 90°)

Square

Rectangle with four equal sides

A diagonal of a polygon is a line segment joining any two nonadjacent vertices.

Area of Quadrilaterals

Square

$$A = s^2$$

s = side length

Rectangle

$$A = l \times w$$

l = length, w = width

Parallelogram

$$A = b \times h$$

b = base, h = height

Trapezoid

$$A = \frac{1}{2}(b_1 + b_2)h$$

b_1, b_2 = bases, h = height

Area is given in square units ($\text{ft}^2, \text{m}^2, \text{mi}^2, \text{cm}^2, \text{in}^2$) | Rhombus: $A = d_1 \times d_2 / 2$ (d_1, d_2 = diagonals)

Examples: Area of Quadrilaterals

Parallelogram

$$b = 9.2 \text{ cm}, h = 6.1 \text{ cm}$$

$$A = b \times h = 9.2 \times 6.1$$

$$A = 56.12 \text{ cm}^2$$

Trapezoid

$$b_1 = 3.6 \text{ ft}, b_2 = 5.5 \text{ ft}, h = 2.9 \text{ ft}$$

$$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(3.6 + 5.5)(2.9)$$

$$A = 13.195 \text{ ft}^2$$

Always label units: area is always in square units (cm^2 , ft^2 , m^2). Forgetting units costs points on the quiz.

Worked Example: Multi-Step Trapezoid

Step 1 — Find the Height

$$b_1 = 3.8, b_2 = 9.2, \text{ slant} = 6.3 \text{ cm}$$

$$d = 9.2 - 3.8 = 5.4; h^2 + 5.4^2 = 6.3^2$$

$$h = \sqrt{10.53} \approx 3.2 \text{ cm}$$

Step 2 — Compute the Area

$$A = \frac{1}{2}(b_1 + b_2)h$$

$$A = \frac{1}{2}(3.8 + 9.2)(3.2) = \frac{1}{2}(13.0)(3.2)$$

$$A \approx 21.1 \text{ cm}^2$$

Key strategy: when the height is not given, look for a right triangle hidden in the trapezoid. Use the Pythagorean theorem to find h first, then apply $A = \frac{1}{2}(b_1 + b_2)h$.

Compound Shapes Strategy

Procedure for Any Compound Shape

Step 1 — Partition

Identify simple shapes within the compound figure (rectangles, triangles, trapezoids).

Step 2 — Label all dimensions

Ensure all necessary measurements are identified for each part.

Step 3 — Compute individual areas

Apply the correct formula to each piece separately.

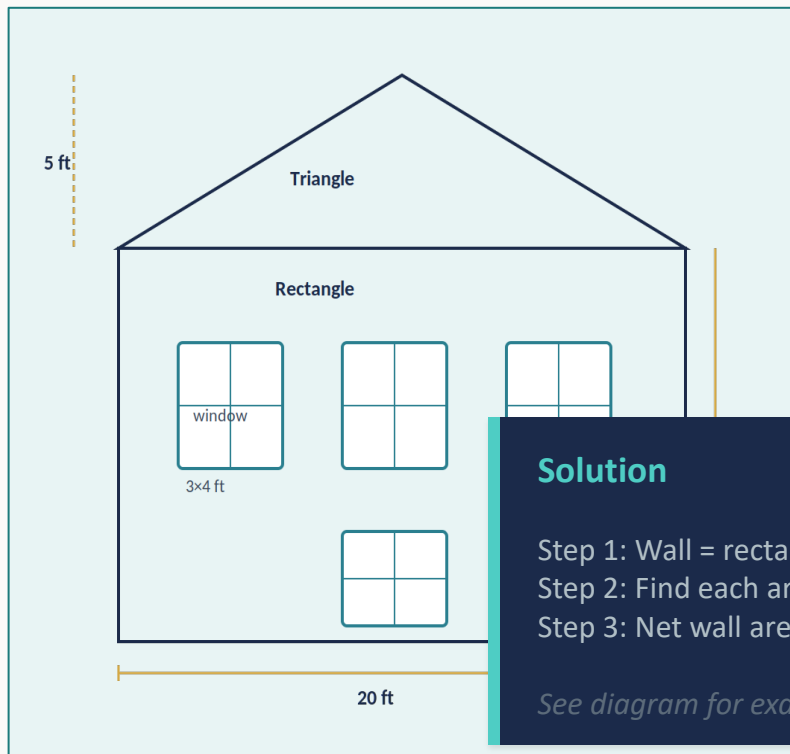
Step 4 — Add or subtract

Add areas of parts (or subtract holes/cutouts).

Step 5 — State units

Always express area in square units (cm^2 , ft^2 , m^2 , etc.).

Application: Area of Compound Shapes



Problem

The wall of a living room with a vaulted ceiling has 4 windows as shown.

If a quart of paint covers 80 ft^2 , how much paint is required to paint these walls?

Solution

Step 1: Wall = rectangle + triangle on top

Step 2: Find each area, then subtract 4 windows

Step 3: Net wall area $\div 80 \text{ ft}^2$ per quart = quarts needed

See diagram for exact dimensions $\rightarrow$ answer varies by wall size.

SECTION 3

Triangles

The most important polygon — every other polygon can be divided into triangles.

Types of Triangles

Scalene

No two sides are equal in length.
No tick marks on sides.

Isosceles

Two sides are equal in length.
Shown with single tick marks on equal sides.

Equilateral

All three sides are equal.
Shown with double tick marks. All angles = 60° .

Right

Has one right angle (90°). The side opposite the right angle is the hypotenuse; the other sides are legs.

Triangle Properties

Sum of Interior Angles

$$\angle A + \angle B + \angle C = 180^\circ$$

The three interior angles of any triangle always sum to exactly 180 degrees.

Perimeter

$$P = a + b + c$$

The total distance around the triangle — sum of all three side lengths.

Area of a Triangle

$$A = \frac{1}{2} \times b \times h$$

A triangle is exactly half of a parallelogram with the same base and height. The height (h) is the perpendicular distance from the base to the opposite vertex.

Triangle Area: Decision Guide

Which Formula to Use?

$A = \frac{1}{2}bh$ — Use when base and height are known

Height must be perpendicular to the base. Works for all triangle types.

$A = (s^2\sqrt{3})/4$ — Use for equilateral triangles

All three sides equal (s). Derived from the 30-60-90 split.

Heron's Formula — Use when only all three sides are known

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = (a+b+c)/2$

No height needed — powerful when height is unknown.

Decision rule: Know height? → $\frac{1}{2}bh$. All equal? → equilateral. Only sides? →

Decision Rule Summary

Know height? → Use $\frac{1}{2}bh$ | All sides equal? → Use equilateral formula | Only sides? → Use Heron's
Heron's is the most versatile — it works for any triangle when all three side lengths are known.

Heron's Formula

Use this when the height isn't known — it only requires the three side lengths. When you know all three side lengths but not the height, use Heron's Formula to find the area.

$$s = (a + b + c) / 2$$

(s = semi-perimeter = half the perimeter)

$$A = \sqrt{s(s - a)(s - b)(s - c)}$$

Example

Triangle with sides $a = 4.5$ mi, $b = 5.6$ mi, $c = 8.2$ mi

$$s = (4.5 + 5.6 + 8.2) / 2 = 18.3 / 2 = 9.15$$

$$s - a = 9.15 - 4.5 = 4.65, \quad s - b = 9.15 - 5.6 = 3.55, \quad s - c = 9.15 - 8.2 = 0.95$$

$$A = \sqrt{9.15 \times 4.65 \times 3.55 \times 0.95} = \sqrt{143.99} \approx 11.87 \text{ mi}^2$$

Equilateral Triangle Formula

Deriving the Formula

For an equilateral triangle with side s :

1. Split into two right triangles
2. Base of each = $s/2$, hypotenuse = s
3. Height: $h^2 + (s/2)^2 = s^2$
 $h^2 = s^2 - s^2/4 = 3s^2/4$
 $h = (s\sqrt{3})/2$
4. $A = \frac{1}{2} \times s \times (s\sqrt{3})/2$

Formula

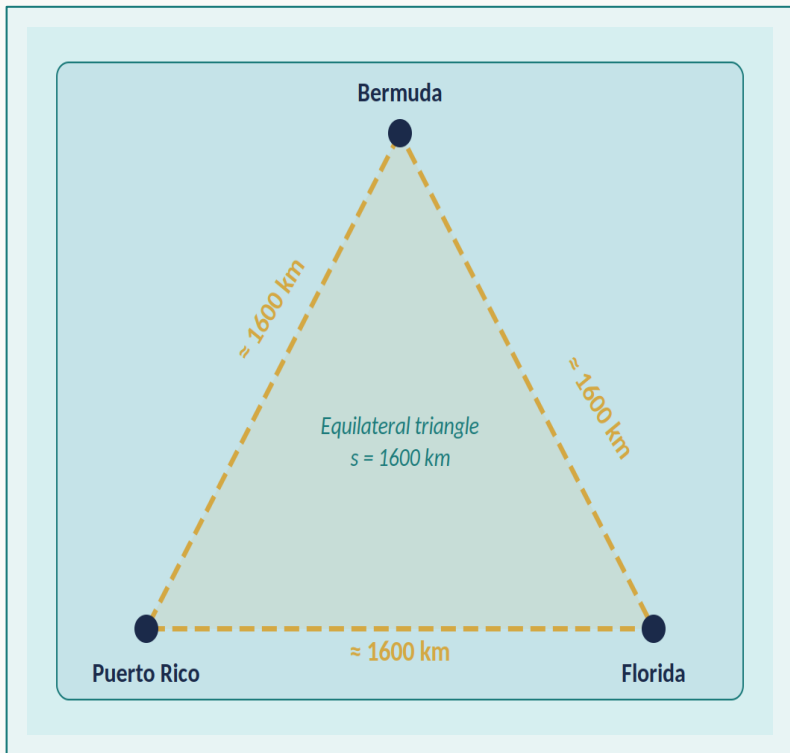
$$A = (s^2\sqrt{3}) / 4$$

$$\text{Height: } h = (s\sqrt{3}) / 2$$

Example: $s = 4.4$ in

$$A = (4.4^2\sqrt{3})/4 \approx 8.39 \text{ in}^2$$

Application: The Bermuda Triangle



Problem

The Bermuda Triangle is sometimes defined as an equilateral triangle 1600 km on a side, with vertices in Bermuda, Puerto Rico, and the Florida coast.

Assuming it is flat, what is its approximate area?

Solution: $1,108,512.5 \text{ km}^2$

Method: $A = (s^2\sqrt{3}) / 4$, with $s = 1600 \text{ km}$
 $= (2,560,000 \times 1.732) / 4 = 4,434,050 / 4 \approx 1,108,512.5 \text{ km}^2$

The Pythagorean Theorem

$$a^2 + b^2 = c^2$$

For a right triangle: the square of the hypotenuse equals the sum of the squares of the two legs.

c = hypotenuse

(longest side, opposite the right angle)

a, b = legs (the two shorter sides)

Checking Right Angles

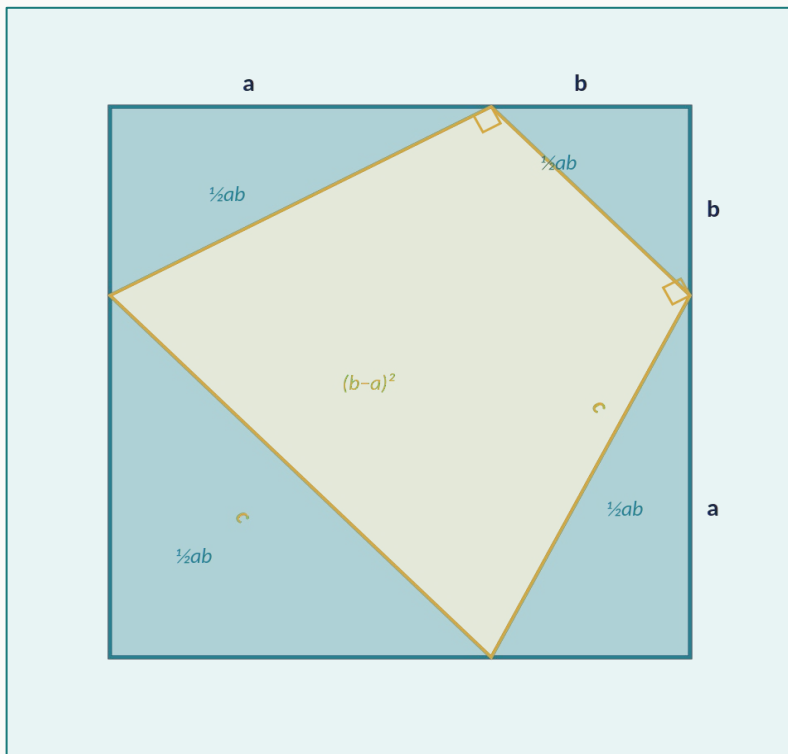
If $a^2 + b^2 = c^2 \rightarrow$ right angle (opposite c)

If $a^2 + b^2 > c^2 \rightarrow$ angle opposite c is acute
(triangle is acute)

If $a^2 + b^2 < c^2 \rightarrow$ angle opposite c is obtuse
(triangle is obtuse)

Example: Box is 13 × 10 in with diagonal 16.75 in. Is it square? $13^2 + 10^2 = 269$, but $16.75^2 = 280.5625$. Since $269 < 280.56$, the angles are obtuse — not right angles.

Pythagorean Theorem: A Geometric Proof



Proof by Area

Large square = 4 triangles + inner square

$$c^2 = 4(\frac{1}{2}ab) + (b-a)^2$$

$$c^2 = 2ab + b^2 - 2ab + a^2$$

$$c^2 = a^2 + b^2 \checkmark$$

The Scarecrow's “Theorem”

From The Wizard of Oz (1939)

The correct theorem ($a^2 + b^2 = c^2$) is **NOT** the Scarecrow's “theorem” from *The Wizard of Oz* — he said:

“The sum of the square roots of any two sides of an Isosceles triangle is equal to the square root of the remaining side.”

Counterexample:

If sides are 3, 3, and 1: $\sqrt{3} + \sqrt{3} = \sqrt{1}$?

$1.732... + 1.732... = 1$??? **What??**

He got his diploma anyway — but his geometry was wrong.

Quiz Connection

The Pythagorean Theorem states $a^2 + b^2 = c^2$ for ANY right triangle — not just isosceles.

On the quiz: identify the hypotenuse first (opposite the right angle), then apply $a^2 + b^2 = c^2$ correctly.

Error Analysis: Triangle Area

Common Mistakes to Avoid

Mistake 1 — Using the slanted side as height

The height must be perpendicular to the base. A slanted leg is NOT the height unless the triangle is right-angled.

Mistake 2 — Forgetting to square the units

Area is always in square units. $12\text{ cm} \times 8\text{ cm} = 96\text{ cm}^2$, not 96 cm.

Mistake 3 — Misidentifying the hypotenuse

The hypotenuse is always opposite the right angle — the longest side.

Mistake 4 — Applying Heron's with the wrong s

s is the semi-perimeter: $s = (a+b+c)/2$, not $a+b+c$.

SECTION 4

Similar Polygons

Similarity extends triangle properties to proportional reasoning — two shapes with equal angles have predictable side length ratios.

Similar Triangles

Two polygons are similar if they have the same shape (but not necessarily the same size).

Property 1

Corresponding angles are equal in measure.

$$\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$$

Property 2

Corresponding sides are proportional.

$$AB/DE = BC/EF = AC/DF$$

We write $\triangle ABC \sim \triangle DEF$ to denote similarity. Only 2 pairs of equal angles are needed to prove similarity.

Quiz Tip

Given $\triangle MNP \sim \triangle QRS$, use corresponding angles to find missing angles (they sum to 180°), and set up proportions like $MN/QR = NP/RS = MP/QS$ to find missing sides.

Similarity: Strategy

Procedure for Solving Similarity Problems

Step 1 — Match corresponding angles

Confirm AA (Angle-Angle) similarity. Order of vertices in the statement matters.

Step 2 — Determine the scale factor

Scale factor = (side of one figure) / (corresponding side of other figure).

Step 3 — Set up the proportion

$a/b = c/d$ — corresponding sides in the same ratio.

Step 4 — Solve for the unknown

Key Reminders

Common shortcut: If two angles are known equal, the third must be equal too (angle sum = 180°).

Scale factor > 1 : figure is enlarging. Scale factor < 1 : figure is shrinking. Always state which direction.

Example: Similar Triangles

Given: $\triangle MNP \sim \triangle QRS$, $\angle P = 41^\circ$, $\angle R = 85^\circ$, $MN = 35$, $QR = 20$, $RS = 24$, $MP = 56$

Find: $\angle N$, $\angle M$, NP , QS

Angles

$\angle N$ corresponds to $\angle R = 85^\circ$

$$\angle N = 85^\circ$$

$$\angle M + \angle N + \angle P = 180^\circ$$

$$\angle M + 85^\circ + 41^\circ = 180^\circ$$

$$\angle M = 54^\circ$$

Sides

$$\text{Scale: } MN/QR = 35/20 = 7/4$$

$$NP/RS = 7/4 \rightarrow NP/24 = 7/4$$

$$NP = 42$$

$$MP/QS = 7/4 \rightarrow 56/QS = 7/4$$

$$QS = 32$$

Applications: Similar Triangles

Shadow Problem

A 6 ft man stands 10 ft from a flagpole. His 8 ft shadow ends at the same point as the flagpole's shadow.

$$6/h = 8/18 \rightarrow h = 13.5 \text{ ft}$$

(Total shadow = 8 + 10 = 18 ft)

River Width Problem

$BC \parallel DE$. $AB = 50$ ft, $AD = 180$ ft, $BC = 60$ ft. Find DE (river width).

$$AB/AD = BC/DE$$

$$50/180 = 60/DE \rightarrow DE = 216 \text{ ft}$$

Strategy: Identify the two similar triangles $\rightarrow$ match corresponding sides $\rightarrow$ set up the proportion $\rightarrow$ cross-multiply and solve.

Application: Parallel Segments

Problem

Two parallel lines are cut by two transversals. Segments $AB = 9$, $BC = 6$, and $DE = 12$. Find EF .

Solution

Parallel lines cut transversals proportionally:

$$AB / BC = DE / EF$$

$$9 / 6 = 12 / EF$$

$$EF = (12 \times 6) / 9 = 72 / 9 = 8$$

Answer: $EF = 8$

Strategy: Parallel lines always divide transversals in the same ratio. Identify corresponding segments before setting up the proportion.

Error Analysis: Similarity

Common Mistakes in Similarity Problems

Mistake 1 — Wrong order in similarity statement

$\triangle ABC \sim \triangle DEF$ means $A \leftrightarrow D$, $B \leftrightarrow E$, $C \leftrightarrow F$. Swapping letters changes which sides correspond.

Mistake 2 — Inverting the ratio

If $AB/DE = BC/EF$, do NOT flip one side: $AB/DE \neq EF/BC$.

Mistake 3 — Mixing sides from different triangles

Always keep numerators from one figure and denominators from the other.

Mistake 4 — Confusing the scale factor direction

Self-Check Strategy

Quick check: Write out the similarity statement first, then list corresponding pairs before setting up any ratio.

If your answer gives a scale factor < 0 or makes sides negative, you have a ratio error — flip and retry.

SECTION 5

Degrees, Minutes & Seconds

Navigation, surveying, and GPS express angles in degrees-minutes-seconds — here we connect that real-world notation to our angle work.

Degrees, Minutes & Seconds

$1^{\circ} = 60 \text{ minutes (60')}$

$1' = 60 \text{ seconds (60'')}$

$1^{\circ} = 3600''$

Reading DMS Notation

$45^{\circ} 30' 15''$

45 = degrees (whole number)

30' = 30 minutes = 30/60 of a degree

15'' = 15 seconds = 15/3600 of a degree

= 45.504167° in decimal form

Where DMS Is Used

- Navigation & maps (latitude/longitude)
- Astronomy (star positions)
- Aviation approach plates
- Surveying & GPS coordinates

Conversions are on the next two slides.

Angles in navigation, surveying, and GPS use degrees-minutes-seconds (DMS) — here's how to convert. Used in navigation, latitude/longitude, and aviation approach plates.

Decimal Degrees → DMS

Procedure: Decimal → DMS

Step 1 — The whole number part is the degrees

Step 2 — Multiply the decimal remainder by 60 → minutes

Step 3 — Multiply that decimal remainder by 60 → seconds

Worked Example: Convert 135.721°

Degrees: 135°

Minutes: $0.721 \times 60 = 43.26 \rightarrow 43'$

Seconds: $0.26 \times 60 = 15.6 \rightarrow 15.6''$

Answer: $135^\circ 43' 15.6''$

DMS → Decimal Degrees

Procedure: DMS → Decimal

Step 1 — Convert minutes: divide by 60

Step 2 — Convert seconds: divide by 3600

Step 3 — Add all three parts together

Worked Example: Convert $120^{\circ} 48' 50''$

Degrees: 120

Minutes: $48 / 60 = 0.8^{\circ}$

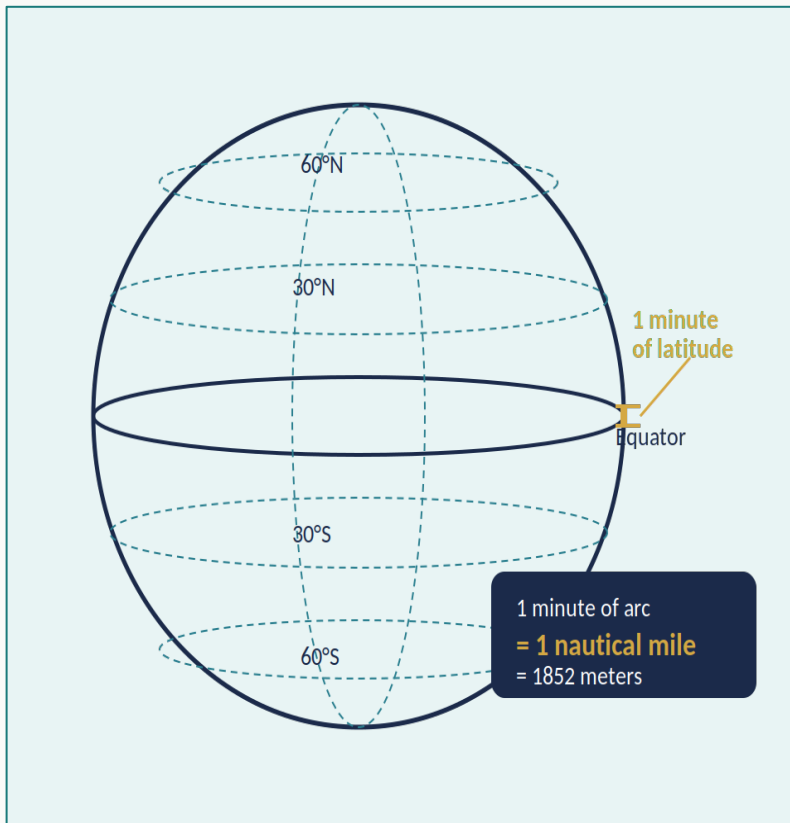
Seconds: $50 / 3600 \approx 0.01389^{\circ}$

Total: $120 + 0.8 + 0.01389 \approx 120.814^{\circ}$

Answer: $\approx 120.814^{\circ}$

Common mistake: forgetting to add all three parts. The answer is $120 + 0.8 + 0.01389$, not just 120.01389 or 0.81389 .

Angles in Navigation: The Nautical Mile



Definition

Historically, one **nautical mile** was defined as the distance of **one minute of latitude** (at sea level).

$$1' = 60''$$

If $1' = 1852$ m, then:

$$1'' = 1852 / 60 \approx 31 \text{ m}$$

Today's official definition (since 1929):

1 nautical mile = 1852 m (exactly)

Practical example:

A ship traveling 10 nautical miles covers $10 \times 1852 = 18,520$ m ≈ 18.5 km

Core Geometric Competencies

By the end of this unit, you should be able to do all of the following:

Angle Reasoning

Classify angles (acute, right, obtuse, straight). Apply vertical, supplementary, and parallel line relationships. Write and solve angle equations.

Polygon Angle Sums

Derive and apply $(n-2) \times 180^\circ$ for interior sums. Use 360° for exterior sums. Find individual angles in regular polygons.

Area Computation

Apply formulas for squares, rectangles, parallelograms, trapezoids, and rhombuses. Partition compound shapes. Choose the correct triangle formula ($\frac{1}{2}bh$, equilateral, Heron's).

Similarity

Identify similar polygons using AA. Write correct similarity statements. Determine scale factors and set up proportions to find missing sides.

DMS Conversion

Convert decimal degrees to DMS (multiply remainders by 60 twice). Convert DMS to decimal (divide by 60 and 3600, then add). Understand the nautical mile connection.

Quiz Preparation: Key Skills

1

Similar Triangles

Find missing angles and sides using proportions ($\triangle MNP \sim \triangle QRS$)

2

Parallel Lines & Angles

Identify corresponding, alternate interior, and alternate exterior angles

3

Triangle Areas

Use $A = \frac{1}{2}bh$, equilateral formula, and Heron's formula

4

Compound Shapes

Break complex shapes into parallelograms and rectangles, sum the areas

5

Polygon Angle Sums

Interior: $(n-2) \times 180^\circ$, Exterior: always 360° , Regular: $(n-2) \times 180^\circ / n$

6

Trapezoid Area

$A = \frac{1}{2}(b_1 + b_2)h$ — identify bases, height, and any diagonal measurements

Remember: Show all work for full credit! Label your units.

Practice Checklist

Before the assessment, make sure you can:

Angle relationships

Identify all parallel line angle pairs by name and state their relationship (equal or supplementary).

Polygon angle sums

Find interior and exterior angle sums for any polygon, and solve for missing angles.

Area computation

Partition compound shapes, apply correct formulas, give answers in square units.

Similarity

Write correct similarity statements, calculate scale factors, set up and solve proportions.

DMS

Convert decimal degrees $\leftrightarrow$ DMS accurately. Understand the nautical mile connection.

MATH 130

Questions? Practice & Review

Worksheet: Covers Sections 1–5 (parallel lines, polygon area, triangles, similar figures, DMS). Due: Next class meeting. Quiz format: 12 problems, no calculator on DMS conversions, calculator permitted for area and similar triangle problems. Show all work for full credit.



Reflection & Questions

Three Prompts for Reflection

Prompt 1

Which formulas require geometric justification rather than just memorization? What is the underlying reasoning?

Prompt 2

Where does proportional reasoning appear most often in this unit? Give two specific examples.

Prompt 3

Which concept required the most careful diagram reading to avoid errors?

Questions? Bring them to the next class or office hours.