

Linear and Angular Velocity: Today

- By the end of class you can:
 - Explain the difference between angular velocity (how fast it turns) and linear velocity (how fast a point moves).
 - Convert rpm to radians per unit of time.
 - Find linear speed with $v = r\omega$, using the radius.
 - Convert the answer to the units asked for (ft/min, mi/h, km/h).
- This week: Quiz 9 (circles) on Wednesday, Test 2 on Friday. Both include speed problems.

Linear and Angular Velocity

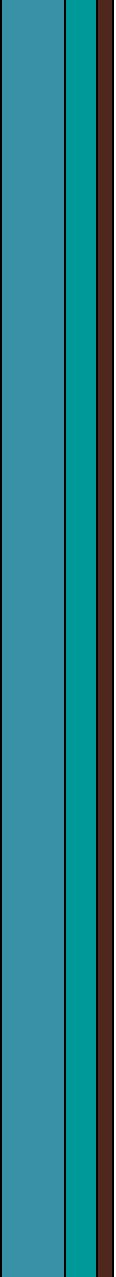
Linear velocity is the change in distance, s , divided by the amount of time elapsed, t :

$$v = s/t$$

- If an object is traveling along a circular arc, the distance traveled is the arc length, $s = r\theta$.
- So, we have **linear velocity** (along the arc) as

$$v = \frac{s}{t} = \frac{r\theta}{t} = r \frac{\theta}{t}$$

- Note that $\frac{\theta}{t}$ is the amount of angle change in time t .
- $\omega = \frac{\theta}{t}$ is called the **angular velocity**.



Key Idea: Same Turn, Different Speed

- Every point on a spinning wheel turns through the same angle in the same time, so every point has the same angular velocity ω .
- A point farther from the center travels a bigger circle in that time, so it moves faster.
- That is why $v = r\omega$: double the radius, double the linear speed.
- Example: on a merry-go-round, a rider at the outer edge moves faster than a rider near the center, even though both go around once per turn.
- Units: ω in radians per unit of time $\times$ r in a length unit gives length per unit of time. Radians have no units.

Linear and Angular Velocity

$$v = \frac{s}{t} = \frac{r\theta}{t} = r \frac{\theta}{t} = r \omega$$

- Linear velocity is angular velocity multiplied by r for an object traveling around a circle of radius r .
 - ω is typically in radians per unit of time (but can be converted to degrees per unit of time)
 - In practice, angular velocity may be stated in another form, such as rpm , but must be converted to radians per unit of time to use the formula.
 - If angular velocity is given in rpm , we can find linear velocity by **converting units** and using the circumference for one rotation (examples follow).

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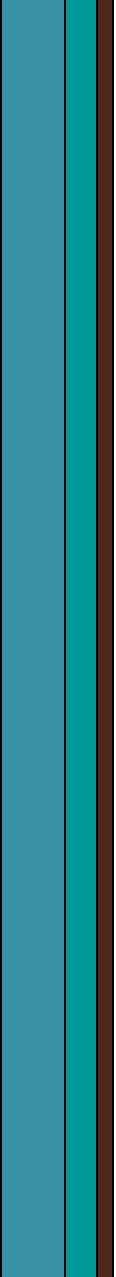
Example:

- A person on a hang glider is moving in a horizontal circular arc of radius 90.0 m with an angular velocity of 0.125 rad/s . What is the person's linear velocity?
- Solution:

$$v = r \omega$$

$$v = (90.0 \text{ m/rad})(0.125 \text{ rad/s}) = 11.25 \text{ m/s. (40.5 km/h)}$$

- (Radians have no units, so $(\text{m})(\text{rad/s}) = \text{m/s}$.)



Strategy: The Unit Chain

- 1. Radius first. If you are given a diameter, divide by 2.
- 2. Turn the rotation rate into radians per unit of time: 1 revolution = 2π rad.
- 3. Multiply: $v = r\omega$.
- 4. Convert units with factors that equal 1: 12 in = 1 ft, 5280 ft = 1 mi, 60 min = 1 h, 1000 m = 1 km.
- 5. Round only at the end, and label the units.
- Shortcut for rpm: $v = (\text{rpm})(2\pi r)$ gives distance per minute.

Examples

- The paddles of a riverboat have a radius of 8.50 ft and turn at 20.0 revolutions (or rotations) per min (rpm). What is the speed of a tip of one of the paddles?

Solution: in your notes



$$v = r \frac{\theta}{t} = 8.5\text{ft} \frac{20(2\pi)}{\text{min}} = \frac{1068.1 \text{ ft}}{\text{min}}$$

$$\frac{2\pi(8.5)\text{ft}}{1 \text{ rotation}} \times \frac{20 \text{ rotations}}{1 \text{ min}} = \frac{1068.1 \text{ ft}}{\text{min}}$$

Examples

- After the brake was applied, a bicycle wheel went through 1.60 rotations before stopping.

Through how many radians did a spoke rotate?

Solution:

$$1.6 \text{ rotations} \cdot \frac{2\pi \text{ rad}}{1 \text{ rotation}} = 3.2\pi = 10.1 \text{ rad}$$

- Through how many radians does the minute hand of a clock move in 25 minutes?

Solution:

$$\frac{25}{60} \text{ rotations} \cdot \frac{2\pi \text{ rad}}{1 \text{ rotation}} = \frac{5\pi}{6} = 2.6 \text{ rad}$$

Examples

- A *mil* is a unit of angle measurement used in artillery defined as a central angle of a circle that intercepts an arc equal in length to $1/6400$ of the circumference.
- How many *mils* are in a central angle of 22.4° .

Solution:

Convert from *degrees* to *mils* by multiplying by *mil* and dividing by *degrees*.

$$22.4^\circ \left(\frac{6400 \text{ mil}}{360^\circ} \right) = 398.2 \text{ mil}$$

Linear and Angular Velocity

$$v = \frac{s}{t} = \frac{r\theta}{t} = r \frac{\theta}{t} = r\omega$$

Example: Control-line Model Aircraft

- A model aircraft on a **17.69 m** line completes **9 laps** in **10.74 sec**. What is the linear speed of the aircraft?
- Solution:

$$\begin{aligned} r\omega &= 17.69\text{m} \frac{9 \text{ rev}}{10.74 \text{ sec}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \\ &= 17.69\text{m} \frac{18\pi \text{ rad}}{10.74 \text{ sec}} = 93.14 \text{ m/sec} \end{aligned}$$



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Examples

- An interstate route exit is a circular arc 425 m long with a central angle of 75.4° .
What is the radius of arc of the exit?

$$s = r\theta$$

$$425 \text{ m} = r \left(75.4^\circ \frac{\pi}{180^\circ} \right)$$

$$r = 323.0 \text{ m}$$

Or,

$$s = \frac{\theta}{360^\circ} (2\pi r)$$

$$425 \text{ m} = \frac{75.4^\circ}{360^\circ} (2\pi r)$$

$$r = 323.0 \text{ m}$$

Solution: in your notes

Examples

- The sweep second hand of a watch is **11.0 mm** long. What is the linear velocity of the tip?

$$v = r \frac{\theta}{t} = 11.0 \text{ mm} \left(\frac{2\pi}{1\text{min}} \right) = 22\pi \frac{\text{mm}}{\text{min}} \approx 69.1 \frac{\text{mm}}{\text{min}}$$

- A vinyl LP record has a diameter of **12 in.** and rotates at **$33\frac{1}{3}$ rpm.**

What is the linear velocity of a point on the outer edge?

$$v = (6 \text{ in}) \left(\frac{100 \text{ rot}}{3 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rot}} \right) = 400\pi \frac{\text{in}}{\text{min}} \approx 1256.6 \frac{\text{in}}{\text{min}}$$

or
$$v = \frac{100 \text{ rot}}{3 \text{ min}} \cdot 2\pi(6) \frac{\text{in}}{\text{rot}} = 400\pi \frac{\text{in}}{\text{min}} \cdot \frac{1\text{ft}}{12\text{in}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 2000\pi \frac{\text{ft}}{\text{hr}}$$

Solution: in your notes

Examples

- The flywheel of an engine is 0.36 m in diameter.
If it is rotating at 750 rot/min (rpm), through what distance does a point on the rim move in 2.00 s?

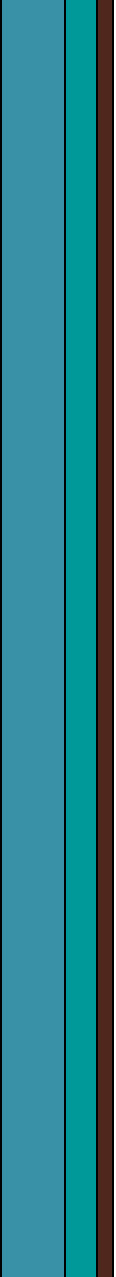
$$v = 0.18 \text{ m} \cdot \frac{750 \text{ rot}}{\text{min}} \cdot \frac{2\pi}{\text{rot}} = 848.2 \frac{\text{m}}{\text{min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 14.137 \frac{\text{m}}{\text{sec}}$$

So it moved $2(14.137) = 28.27 \text{ m}$

- A drill bit $\frac{1}{2}$ in. in diameter rotates at 1800 rpm.
What is the linear velocity of a point on its circumference?

$$v = \frac{1}{4} \text{ in} \frac{1800 \text{ rot}}{\text{min}} \cdot \frac{2\pi}{\text{rot}} = 900\pi \frac{\text{in}}{\text{min}} = 2827 \frac{\text{in}}{\text{min}}$$

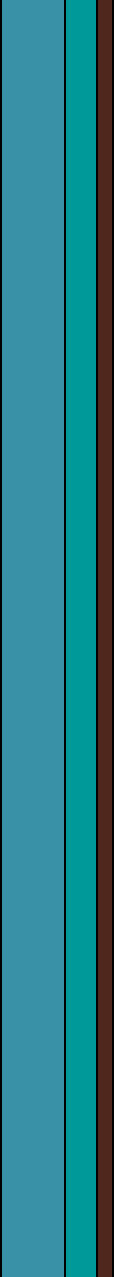
Solution: in your notes



Examples

- Through what total angle does the drive shaft of a car rotate in 1 s when the tachometer reads 3200 rpm?

Solution: in your notes



Solution: Drive Shaft

- 3200 rev/min means $3200 \div 60 \approx 53.3$ revolutions in 1 second.
- In radians: $53.3 \text{ rev} \times 2\pi \text{ rad/rev} \approx 335 \text{ rad}$.
- In degrees: $53.3 \text{ rev} \times 360^\circ \text{ per rev} = 19,200^\circ$.
- Check: $3200 \text{ rpm} \times 2\pi \div 60 = 335 \text{ rad/s}$, so 335 rad in 1 s.

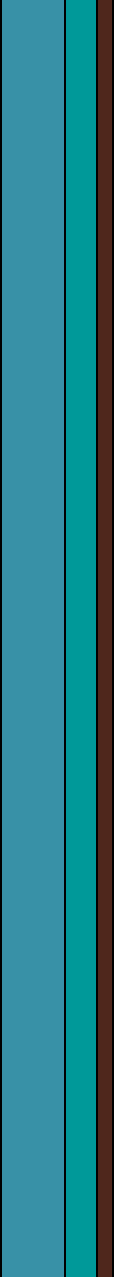
Examples

The blades on the GE Model 1.5s, 1.5 Mw wind turbine (windmill) are 116 ft long.

- What is the linear velocity the tip of the blade, if the windmill rotates at 22.0 r/min?
- What is the surface area of the rotating blades measured in acres? (1 acre = 43,560 ft²)

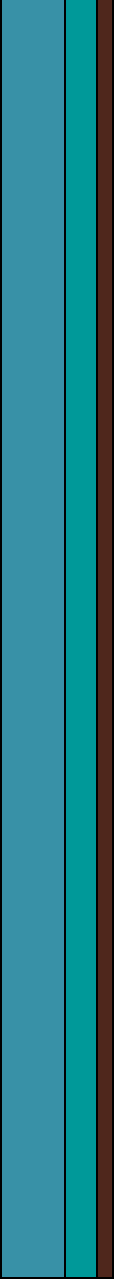
Solution: in your notes

- Wind Fast Fact #17: As of May 2016, the United States is home to 48,500 operating wind turbines. These turbines are present across 40 states plus Puerto Rico. - <http://windenergyfoundation.org/>
- Common Industrial Wind Turbines: <http://www.aweo.org/windmodels.html>
- http://www.awea.org/Issues/Content.aspx?ItemNumber=832&gclid=CjwKEAjwt8ZzHBRCUwrrV59XinXUSJADSTE5kPm_mWpkB7_JfzrycKdFeDjg7aQ9J3BV5weUCtXgxHBoCFu7w_wcB
- <http://savetheeaglesinternational.org/new/us-windfarms-kill-10-20-times-more-than-previously-thought.html>



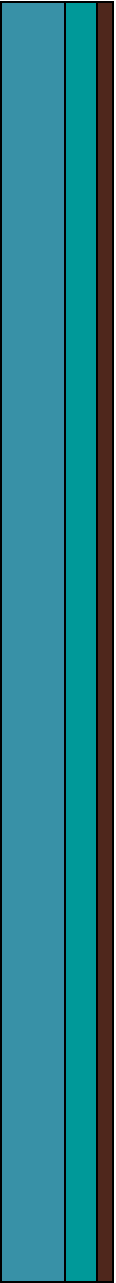
Solution: Wind Turbine

- The blade length is the radius: $r = 116$ ft.
- $\omega = 22.0 \text{ rev/min} \times 2\pi \text{ rad/rev} = 44\pi \text{ rad/min}$.
- $v = r\omega = 116(44\pi) \approx 16,035 \text{ ft/min}$.
- In mi/h: $16,035 \text{ ft/min} \times 60 \text{ min/h} \div 5280 \text{ ft/mi} \approx 182.2 \text{ mi/h}$.
- Area swept by the blades: $A = \pi r^2 = \pi(116)^2 \approx 42,273 \text{ ft}^2$.
- In acres: $42,273 \div 43,560 \approx 0.970 \text{ acre}$.



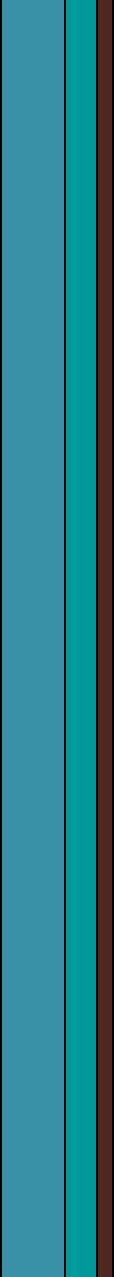
Common Errors

- Using the diameter in $v = r\omega$. This doubles the answer.
- Leaving ω in rpm or in degrees. Convert to radians per unit of time first.
- Rounding in the middle. Keep full values until the last step.
- Mixing time units: seconds in one factor, minutes in another.
- Leaving off units. A speed needs a distance unit and a time unit.



You Try (Worksheet Problems 11 and 12a)

- A Cessna 310 propeller has diameter 81 in and turns at 2200 rpm. Find the linear speed of the tip in ft/min and mi/h.
- The sweep second hand of a watch is 21.0 mm long. Find the linear velocity of the tip in mm/s and cm/min.
- Use the unit chain: radius, radians per time, $v = r\omega$, convert, round.



Exit Ticket

- A box fan has blades 24 in across (diameter) and turns at 1500 rpm.
- a) What is the radius in feet?
- b) What is the linear speed of a blade tip in ft/min?
- c) What is that speed in mi/h?
- Show the unit chain and round only at the end.