

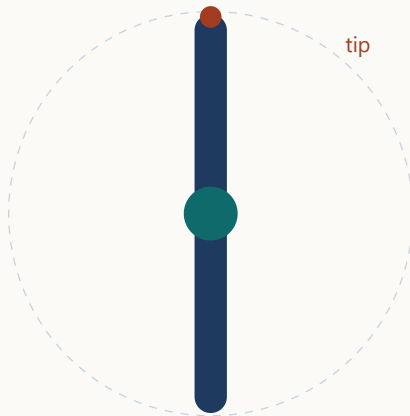
How fast is the tip of a propeller?

A Cessna 310 propeller is **81 inches** across and turns at **2200 rpm**.

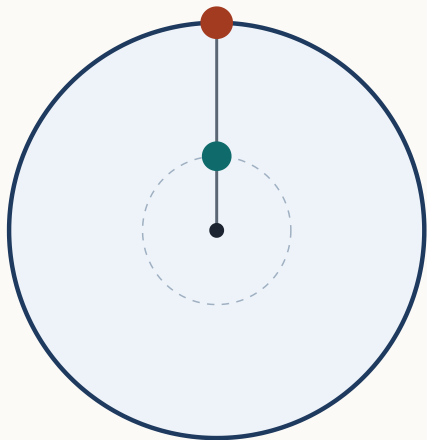
Guess: how fast is the tip moving, in miles per hour?

About **530 mph**.

By the end of class you can get that number yourself, and do it on Quiz 9 (Wednesday) and Test 2 (Friday).



Same turn, different speed



- Both dots turn through the **same angle** in the same time: the same **angular velocity** ω .
- The outer dot travels a **bigger circle** in that time, so it moves faster: a bigger **linear velocity** v .
- ω = how fast it **turns** (radians per unit of time).
 v = how fast a point **moves** (distance per unit of time).

Predict

Ana sits **2 m** from the center of a merry-go-round. Ben sits **6 m** from the center. It turns once every **10 s**.

Question 1

Who has the larger angular velocity?

Neither. Both turn 2π rad in 10 s: $\omega = 2\pi/10 \approx 0.628$ rad/s.

Question 2

Who has the larger linear velocity, and how many times larger?

Ben, 3 times larger: 6 m is 3×2 m. (Ana ≈ 1.26 m/s, Ben ≈ 3.77 m/s.)

Where $v = r\omega$ comes from

$$v = s/t$$

On a circle, the distance traveled is an arc: $s = r\theta$ (θ in radians).

$$v = r\theta/t = r \cdot \theta/t = r\omega$$

Linear velocity = radius $\times$ angular velocity, with ω in **radians** per unit of time. Radians have no units, so $(\text{ft})(\text{rad}/\text{min}) = \text{ft}/\text{min}$.

The unit chain: five lines, every time

- 1 **Radius.** Given a diameter? Divide by 2.
- 2 **Angular velocity in radians.** $\text{rpm} \times 2\pi \text{ rad per revolution}$.
- 3 **Multiply.** $v = r\omega$
- 4 **Convert units.** $12 \text{ in} = 1 \text{ ft} \cdot 5280 \text{ ft} = 1 \text{ mi} \cdot 60 \text{ min} = 1 \text{ h} \cdot 1000 \text{ m} = 1 \text{ km} \cdot 3600 \text{ s} = 1 \text{ h}$
- 5 **Round once, at the end,** and write the units.

Shortcut for rpm: $v = (\text{rpm})(2\pi r)$ gives distance per minute. It is the same as lines 2 and 3 together.

WORKED EXAMPLE

Riverboat paddle wheel

The paddles have a radius of **8.50 ft** and turn at **20.0 rpm**. How fast is a paddle tip moving, in ft/min and mi/h?

1 · Radius $r = 8.50 \text{ ft}$

2 · Radians per minute $\omega = 20.0 \text{ rev/min} \cdot 2\pi \text{ rad/rev} = 40\pi \text{ rad/min}$

3 · Multiply $v = r\omega = 8.50(40\pi) = 340\pi \approx 1068.1 \text{ ft/min}$

4–5 · Convert, round $1068.1 \text{ ft/min} \cdot 60 \text{ min/1 h} \cdot 1 \text{ mi/5280 ft} \approx 12.1 \text{ mi/h}$

Hang glider

A hang glider flies a circular arc of radius **90.0 m** with angular velocity **0.125 rad/s**. Find its speed in m/s and km/h.

1 · Radius $r = 90.0 \text{ m}$

2 · Radians per second $\omega = 0.125 \text{ rad/s}$ (already in radians)

3 · Multiply $v = r\omega = 90.0(0.125) = 11.25 \text{ m/s}$

4–5 · Convert, round $11.25 \text{ m/s} \cdot 3600 \text{ s/1 h} \cdot 1 \text{ km/1000 m} = 40.5 \text{ km/h}$ Round 11.25 to 11.3 first and you get 40.7, which is wrong.

Back to the propeller

Diameter **81 in**, turning at **2200 rpm**. Find the tip speed in ft/min and mi/h.

1 · Radius (done for you) $r = 81 \div 2 = 40.5$ in

2 · Radians per minute $\omega = \text{-----} = 2200 \cdot 2\pi = 4400\pi$ rad/min

3 · Multiply $v = \text{-----}$ in/min $= 40.5(4400\pi) \approx 559,832$ in/min

4–5 · Convert, round ----- ft/min and ----- mi/h $\approx 46,652.7$ ft/min ≈ 530.1 mi/h

SPOT THE ERROR

Three wrong answers, one right one

A wheel **2.0 ft** across turns at **300 rpm**. Find the rim speed in ft/min.

Answer A

$$v = 2.0(300)(2\pi) \approx 3769.9 \text{ ft/min}$$

Used the diameter. The answer is doubled.

Answer B

$$v = 1.0(300)(360) = 108,000 \text{ ft/min}$$

Used degrees. ω must be in radians.

Answer C

$$v = 1.0(300) = 300 \text{ ft/min}$$

Left ω in rpm (forgot 2π per revolution).

Answer D

$$v = 1.0(300)(2\pi) = 600\pi \approx 1885.0 \text{ ft/min}$$

Correct: radius, radians, multiply.

Watch second hand (worksheet 12a)

The sweep second hand of a watch is **21.0 mm** long. It makes one turn per minute.

Find the linear velocity of its tip in **mm/s** and in **cm/min**. Use the five lines.

- 1 Radius
- 2 Radians per second
- 3 Multiply
- 4 Convert
- 5 Round and label

Watch second hand: answer

1 · Radius $r = 21.0 \text{ mm}$

2 · Radians per second $\omega = 2\pi \text{ rad} / 60 \text{ s} = \pi / 30 \text{ rad/s}$

3 · Multiply $v = 21.0 \cdot \pi / 30 = 0.7\pi \approx 2.2 \text{ mm/s}$

4–5 · Convert, round $0.7\pi \text{ mm/s} \cdot 60 \text{ s} / 1 \text{ min} \cdot 1 \text{ cm} / 10 \text{ mm} \approx 13.2 \text{ cm/min}$

Wind turbine

The blades of a GE 1.5s wind turbine are **116 ft** long and turn at **22.0 rpm**.

a) Tip speed in ft/min and mi/h

$r = 116$ ft (the blade is the radius)

$$v = 116(22.0)(2\pi) = 5104\pi \approx 16,035$$

ft/min

$$\approx 182.2 \text{ mi/h}$$

b) Area swept by the blades, in acres (1 acre = 43,560 ft²)

$$A = \pi r^2 = \pi(116)^2 \approx 42,273 \text{ ft}^2$$

$$42,273 \div 43,560 \approx 0.970 \text{ acre}$$

This week

Wednesday

Quiz 9: circles, in class on paper

- degrees to radians
- circumference, arc length, sector area
- central and inscribed angles
- a volume word problem

Practice: Quiz 9 practice (PDF) on the course page

Friday

Test 2, in class on paper

- Printed: compound and continuous interest, Heron's formula
- **From memory:** $v = r\omega$, $\text{rpm} \times 2\pi$, arc length, sector area, circle formulas

Practice: Test 2 practice (PDF) and the Test 2 review page

Box fan

A box fan's blades are **24 in** across and turn at **1500 rpm**.

a) What is the radius, in feet?

b) What is the linear speed of a blade tip, in ft/min?

c) What is that speed in mi/h?

Show all five lines of the unit chain. Round only at the end.