Math 114 Module 5: Finances and Interest

Prof. Volk

Spring 2025

Point Total after Module 4

Test (125 points)
Project (25 points)
HWs 1-4 (80 points)
Quizzes 1-3 (30 points)
260 points = 100%

*10 points bonus for CRC
*5 points bonus for Bio

A	234-260
B	208-233
C	182 -207

What to do in lecture?

Show up

Take notes

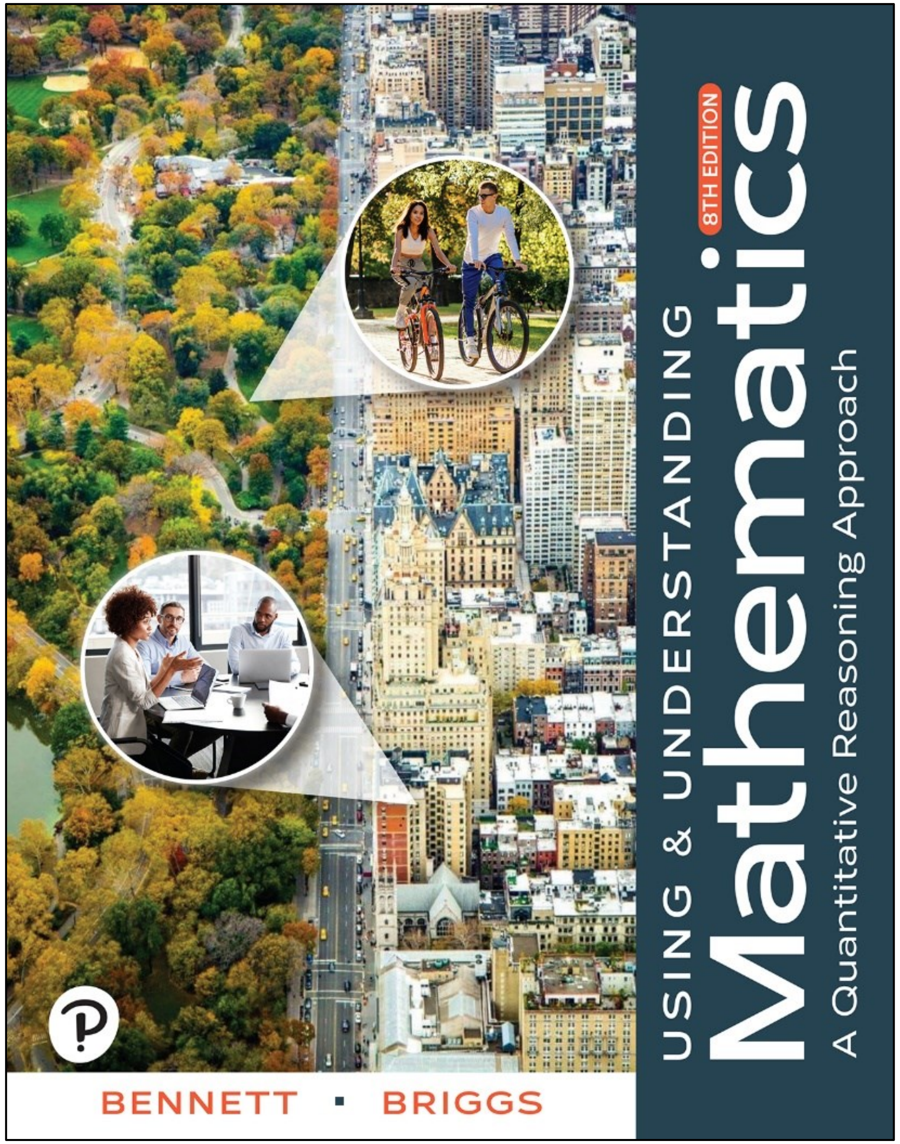
Key formulas

New words

Hints and shortcuts

Answer questions

Make Progress



This Week's Reading

Chapter 4

(Sections A, B, & C)

Managing Money

Homework 5: 25 Questions

Due Tuesday 11:59 pm

Questions 1 & 3	• Give examples and explain concepts and terms related to budgets, expenses, and insurance.
Question 2 & 4	• Decide if statements related to personal finance are clearly true.
Questions 5 & 6	• Solve applications involving calculating the cost per year of expenses.
Questions 7-10	• Solve applications involving comparing spending and wage patterns.
Questions 11 & 12	• Solve applications involving finding total expenses, least expensive options, or most profitable options.
Questions 13-15	• Calculate the accumulated balance or interest for accounts earning simple interest or compound interest.
Question 16 & 17	• Decide if statements related to compounding interest and percentage rates are clearly true.
Question 18	• Evaluate expressions using the four basic rules of algebra.
Questions 19 & 20	• Find the present value for an account.
Question 21 & 22	• Find the time it will take for the balance in an account to reach a given amount.
Questions 23 & 24	• Calculate total and annual returns on investments.
Question 25	• Calculate and interpret price-to-earning ratios.

Quiz 5: 10 Questions

Due Wednesday 11:59 pm

Question 1	Solve applications involving calculating the cost per year of expenses.
Question 2	Solve applications involving calculating interest payments.
Questions 3	Solve applications involving calculating monthly expenses.
Question 4	Solve applications involving calculating net cash flow.
Questions 5	Calculate the accumulated balance or interest for accounts earning simple interest or compound interest.
Questions 6	Calculate and interpret balances, interest, and deposit amounts using the savings plan formula.
Questions 7	Evaluate expressions involving powers and roots.
Questions 8 & 9	Find the present/future value for an account.
Question 10	Calculate and interpret price-to-earning ratios.

Module 5: **Learning Outcomes**

- Give examples and explain concepts and terms related to budgets, expenses, and insurance.
- Solve applications involving calculating net cash flow and monthly expenses
- Find the present/future value for an account.
- Calculate and interpret price-to-earning ratios.

Ecclesiastes 5:10

10 Whoever loves money never has enough; whoever loves wealth is never satisfied with their income. This too is meaningless.

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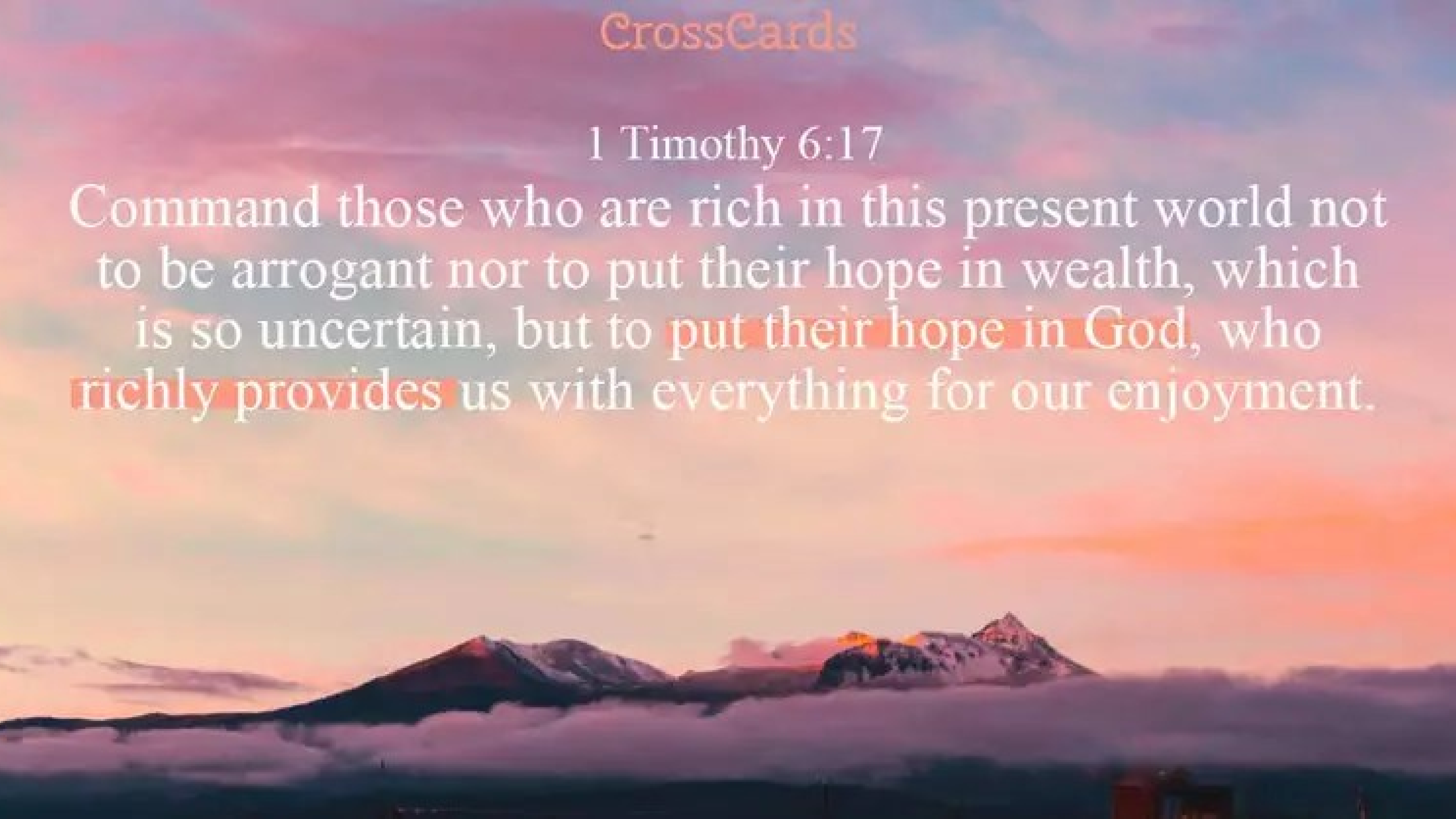
1 Timothy 6:10

10 For the love of money is a root of all kinds of evil. Some people, eager for money, have wandered from the faith and pierced themselves with many griefs.

Controlling Your Finances

- Know your bank balance. Avoid bouncing a check or have a debit card rejected.
- Know what you spend, in particular, keep track of debit and credit card spending.
- Don’t buy on impulse. Think first; buy only if the purchase makes sense.
- Make a budget, and don’t overspend it.


CrossCards
1 Timothy 6:17
Command those who are rich in this present world not to be arrogant nor to put their hope in wealth, which is so uncertain, but to put their hope in God, who richly provides us with everything for our enjoyment.

Example: Credit Card Interest (1 of 2)

Cassidy has begun keeping her spending under better control, but she still can’t fully pay off her credit card balance. She maintains an average monthly balance of about $1100, and her card charges a 24% annual interest rate, which it bills at a rate of 2% per month. How much is she spending on credit card interest?

Solution

Her average monthly interest is 2% of the $1100 average balance.

$$0.02 \times \$1100 = \$22$$

Example: Credit Card Spending (2 of 2)

Multiply by 12 months in a year gives her annual interest payment.

$$12 \times \$22 = \$264$$

Interest alone is costing Cassidy more than $260 per year – a significant amount for someone living on a tight budget. Clearly, she’d be a lot better off if she could find a way to pay off that credit card balance quickly and end those interest payments.

A Four-Step Budget-Making Process

1. Determine your average monthly **income**. Be sure to include an average monthly amount for any income you do not receive monthly (such as once-a-year payments).

2. Determine your average monthly **expenses**. Be sure to include an average amount for expenses that don’t recur monthly, such as expenses for tuition, books, vacations, insurance, and holiday gifts.

3. Determine your **net monthly cash flow** by subtracting your total expenses from your total income.

4. Make adjustments as needed.

Matthew 21:12-13

12 Jesus entered the temple courts and drove out all who were buying and selling there. He overturned the tables of the money changers and the benches of those selling doves.

13 “It is written,” he said to them, “ ‘My house will be called a house of prayer,’but you are making it ‘a den of robbers.’”

Example: College Expenses (1 of 2)

In addition to your monthly expenses, you have the following college expenses that you pay twice a year: $3500 for tuition each semester, $750 in student fees each semester, and $800 for textbooks each semester. How should you handle these expenses in computing your monthly budget?

Solution

Amount paid over a whole year:

$$2 \times (\$3500 + \$750 + \$800) = \$10{,}100$$

Example: College Expenses (2 of 2)

$$2 \times (\$3500 + \$750 + \$800) = \$10{,}100$$

To average this total expense for the year on a monthly basis, we divide by 12.

$$\$10{,}100 \div 12 \approx \$842$$

Your average monthly college expense for tuition, fees, and textbooks comes to a little less than $850, so you should put $850 per month into your expense list.

Example: Cost of a College Class (1 of 2)

Across all institutions, the average cost of a three-credit college class is approximately $1500. Suppose that, between class time, commute time, and study time, the average class requires about 10 hours per week of your time. Assuming that you could have had a job paying $15 per hour, what is the net cost of the class compared to working? Is it a worthwhile expense?

Example: Cost of a College Class (2 of 2)

Solution

A typical college semester lasts 14 weeks, so your "lost" work wages for the time you spend on the class comes to

$$14\text{ wk} \times \frac{10\text{ hr}}{\text{wk}} \times \frac{\$15}{\text{he}} = \$2100$$

We find your total net cost of taking the class by adding this to the $1500 that the class itself costs. The result is $3600. Whether this expense is worthwhile is subjective, but remember college graduates earn nearly $1.3 million more over a career than a high school graduate.

Proverbs 13:11

11 Dishonest money dwindles away, but whoever gathers money little by little makes it grow.

Insurance Costs

The **premium** is the amount you pay to purchase the policy. Premiums are often paid once or twice a year, though sometimes you may pay them more often.

A **deductible** is the amount you are personally responsible for before the insurance company will pay anything.

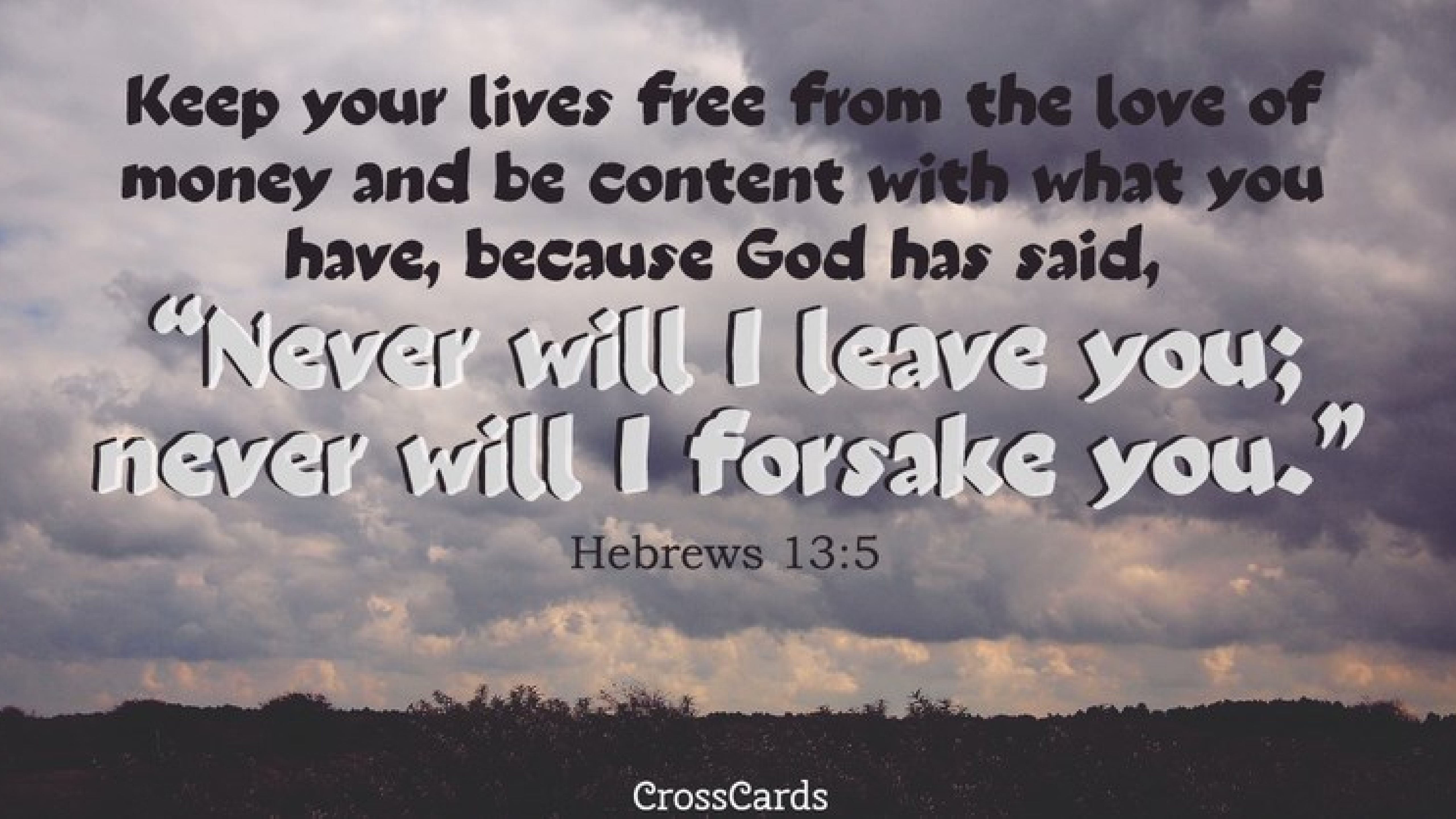
A **co-payment** usually applies to health insurance and is the amount you pay each time you use a particular service that is covered by the insurance policy.

Psalms 37:16-17

16 Better the little that the righteous have than the wealth of many wicked;
17 for the power of the wicked will be broken, but the LORD upholds the righteous.

Definitions

- The **principal** in financial formulas is the balance upon which interest is paid.
- **Simple interest** is interest paid only on the original principal, and not on any interest added at later dates.
- **Compound interest** is interest paid both on the original principal and on all interest that has been added to the original investment.


Keep your lives free from the love of money and be content with what you have, because God has said,
"Never will I leave you; never will I forsake you."
Hebrews 13:5
CrossCards

Example: Savings Bond (1 of 2)

While banks almost always pay compound interest, bonds usually pay simple interest. Suppose you invest $1000 in a savings bond that pays simple interest of 10% per year. How much total interest will you receive in 5 years? If the bond paid compound interest, would you receive more or less total interest? Explain.

Solution

Simple interest: every year you receive the same interest payment.

$$10\% \times 1000 = \$100$$

Example: Savings Bond (2 of 2)

Therefore, you receive a total of \$500 in interest over 5 years.

With compound interest, you receive more than \$500 in interest because the interest each year is calculated on your growing balance rather than your original investment.

Second interest payment:

$$10\% \times \$1100 = \$110$$

This raises your balance faster than simple interest.

Example: Simple and Compound Interest (1 of 3)

Suppose you could invest $100 in two accounts that each pay an interest rate of 10% per year, but one pays simple interest and the other pays compound interest. Make a table to show the growth of each over a 5-year period. Use the compound interest formula to verify the result in the table for the compound interest case.

Example: Simple and Compound Interest (2 of 3)

Compare the growth in a $100 investment for 5 years at 10% simple interest per year and at 10% interest compounded annually.

	Simple Interest Account		Compound Interest Account	
End of Year	Interest Paid	Old Balance + Interest = New Balance	Interest Paid	Old Balance + Interest = New Balance
1	10% × $100 = $10	$100 + $10 = $110	10% × $100 = $10	$100 + $10 = $110
2	10% × $100 = $10	$110 + $10 = $120	10% × $110 = $11	$110 + $11 = $121
3	10% × $100 = $10	$120 + $10 = $130	10% × $121 = $12.10	$121 + $12.10 = $133.10
4	10% × $100 = $10	$130 + $10 = $140	10% × $133.10 = $13.31	$133.10 + $13.31 = $146.41
5	10% × $100 = $10	$140 + $10 = $150	10% × $146.41 = $14.64	$146.41 + $14.64 = $161.05

The compound interest account earns $11.05 more than the simple interest account.

Example: Simple and Compound Interest (3 of 3)

To verify the final entry in the table with the compound interest formula.

$$A = P \times (1 + \text{APR})^{Y}$$

$$= \$100 \times (1 + 0.1)^{5}$$

$$= \$100 \times 1.1^{5}$$

$$= \$100 \times 1.6105$$

$$= \$161.05$$

Compound Interest

Show how quarterly compounding affects a \$1000 investment at 8% per year.

Table: Quarterly Interest Payments

After *N* Quarters	Interest Paid	New Balance
1st quarter (3 months)	$2\% \times \$1000 = \$20$	$\$1000 + \$20 = \$1020$
2nd quarter (6 months)	$2\% \times \$1020 = \$20.40$	$\$1020 + \$20.40 = \$1040.40$
3rd quarter (9 months)	$2\% \times \$1040.40 = \$20.81$	$\$1040.40 + \$20.81 = \$1061.21$
4th quarter (1 full year)	$2\% \times \$1061.21 = \$21.22$	$\$1061.21 + \$21.22 = \$1082.43$

Matthew 19:21

21 Jesus answered, “If you want to be perfect, go, sell your possessions and give to the poor, and you will have treasure in heaven. Then come, follow me.”

Example: Monthly Compounding at 3% (1 of 3)

Suppose you deposit $5000 in a bank account that pays an APR of 3% and compounds interest monthly. How much money will you have after 5 years? Compare this amount to the amount you'd have if interest were paid only once each year.

Solution

The starting principal is P = $5000 and the interest rate is APR = 0.03. Monthly compounding means that interest is paid n = 12 times a year, and we are considering a period of Y = 5 years.

Example: Monthly Compounding at 3% (2 of 3)

$$A = P \times \left(1 + \frac{\text{APR}}{n}\right)^{nY} = \$5000 \times \left(1 + \frac{0.03}{12}\right)^{(12\times5)}$$

$$= \$5000 \times (1.0025)^{60}$$

$$= \$5808.08$$

For interest paid only once each year, we find the balance after 5 years by using the formula for compound interest paid once a year.

$$A = P \times (1 + APR)^{Y} = \$5000 \times (1 + 0.03)^{5}$$

$$= \$5000 \times (1.03)^{5}$$

$$= \$5796.37$$

Example: Monthly Compounding at 3% (3 of 3)

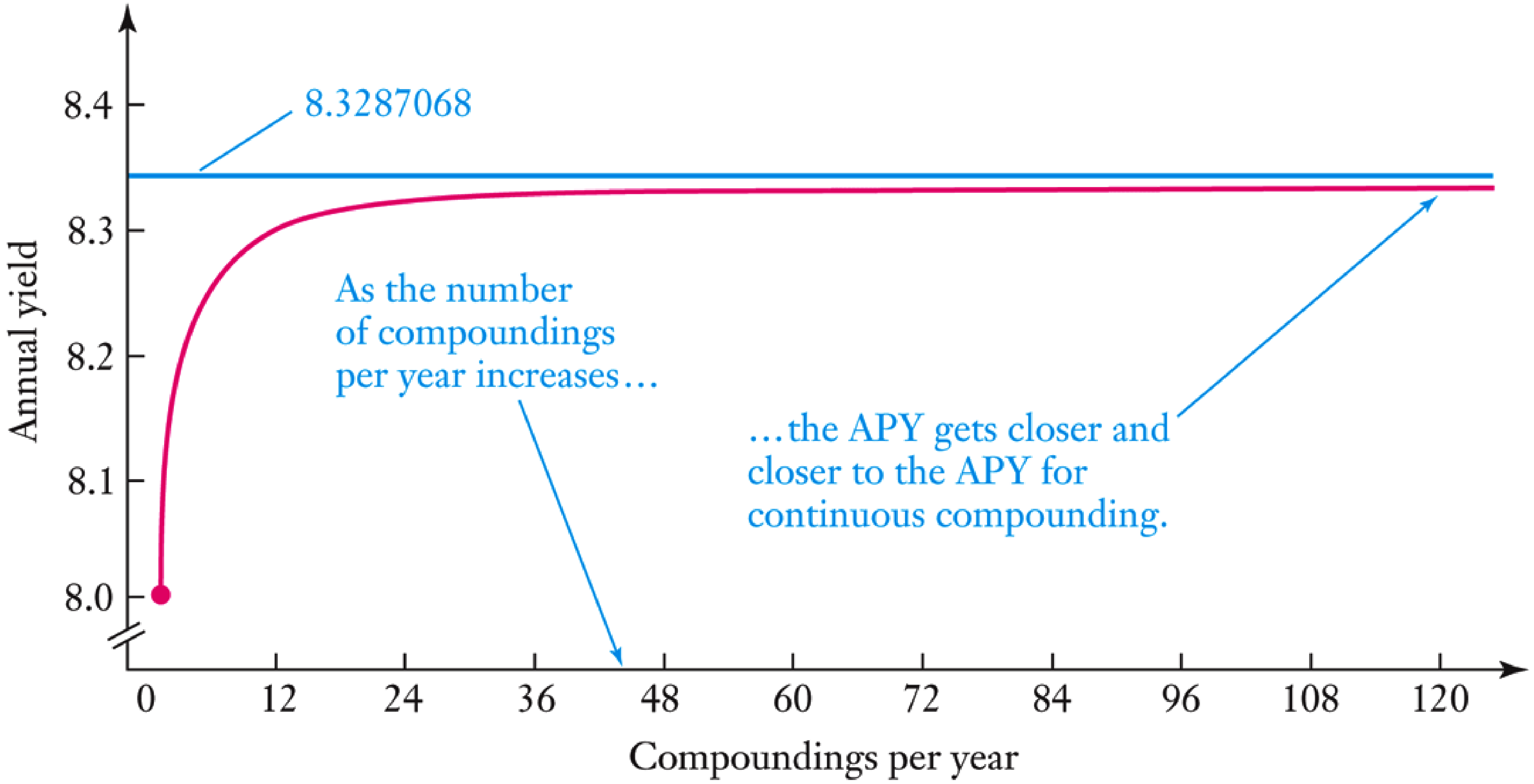
After 5 years, monthly compounding gives you a balance of $5808.08 while annual compounding gives you a balance of $5796.37. That is monthly compounding earns $5808.08 – $5796.37 = $11.71 more, even though the APR is the same in both cases.

Continuous Compounding (1 of 2)

Show how different compounding periods affect the APY for an APR of 8%.

n	APY	*n*	APY
1	8.0000000%	1000	8.3283601%
4	8.2432160%	10,000	8.3286721%
12	8.2432160%	1,000,000	8.3287064%
365	8.2432160%	10,000,000	8.3287067%
500	8.2432160%	1,000,000,000	8.3287068%

Continuous Compounding (2 of 2)



For the following situation, find the average monthly expense that you would use in budgeting for the given expense. Note: Annual means once a year, and semiannual means twice a year.

Prorate the following expenses and find the corresponding monthly expense.

Lan pays a semiannual premium of $700 for automobile insurance, a monthly premium of $170 for health insurance, and an annual premium of $450 for life insurance.

$700
Every 6 months

$170
Every 1 months

$450
Every 12 months

$$\frac{\$700}{6} + \frac{\$170}{1} + \frac{\$450}{12}$$

$$\$116.67 + \$170 + \$37.50$$

$$\boxed{\$324.17}$$

Romans 13:8

8 Let no debt remain outstanding, except the continuing debt to love one another, for whoever loves others has fulfilled the law.

The expenses and income of an individual are given in table form to the right. Find the net monthly cash flow (it may be negative or positive). Assume that amounts shown for salaries and wages are after taxes and that 1 month = 4 weeks.

Income	Expenses
Salary: $35,300/year	House payments: $770/month
Pottery Sales: $196/month	Groceries: $100/week
	Household expenses: $135/month
	Health insurance: $450/month
	Car insurance: $500 twice a year
	Savings plan: $25/month
	Donations: $660/year
	Miscellaneous: $500/month

$$Monthly\ Income = \frac{35300}{12} + 196 = \$3137.67$$

$$Monthly\ Expenses = 770 + 4(100) + 135 + 450 + \frac{500}{6} + 25 + \frac{660}{12} + 500$$

$$Monthly\ Expenses = \$2418.33$$

$$Net\ Monthly\ Cash\ Flow = \$3137.67 - \$2418.33 = \boxed{+719.34}$$

Find the monthly interest payment in the situation below.

Vic bought a new plasma TV for $2300. He made a down payment of $100 and then financed the balance through the store. Unfortunately, he was unable to make the first monthly payment and now pays 5% interest per month on the balance (while he watches his TV).

$$\$2300\ Total\ Cost$$

$$\$100\ Down\ Payment$$

$$\$2200\ Financed$$

$$5\%\ of\ \$2200\ Monthly\ Interest$$

$$\$110\ Monthly\ interest$$

Calculate the $\boxed{\text{amount of money you'll have at the end}}$ of the indicated time period.

You invest $2000 in an account that pays $\boxed{\text{simple interest}}$ of 2% for 20 years.

$$Simple\ Interest\colon\ \$2000 \times 2\% \times 20$$

$$Simple\ Interest\colon \$800$$

$$Future\ Value\colon \$2800$$

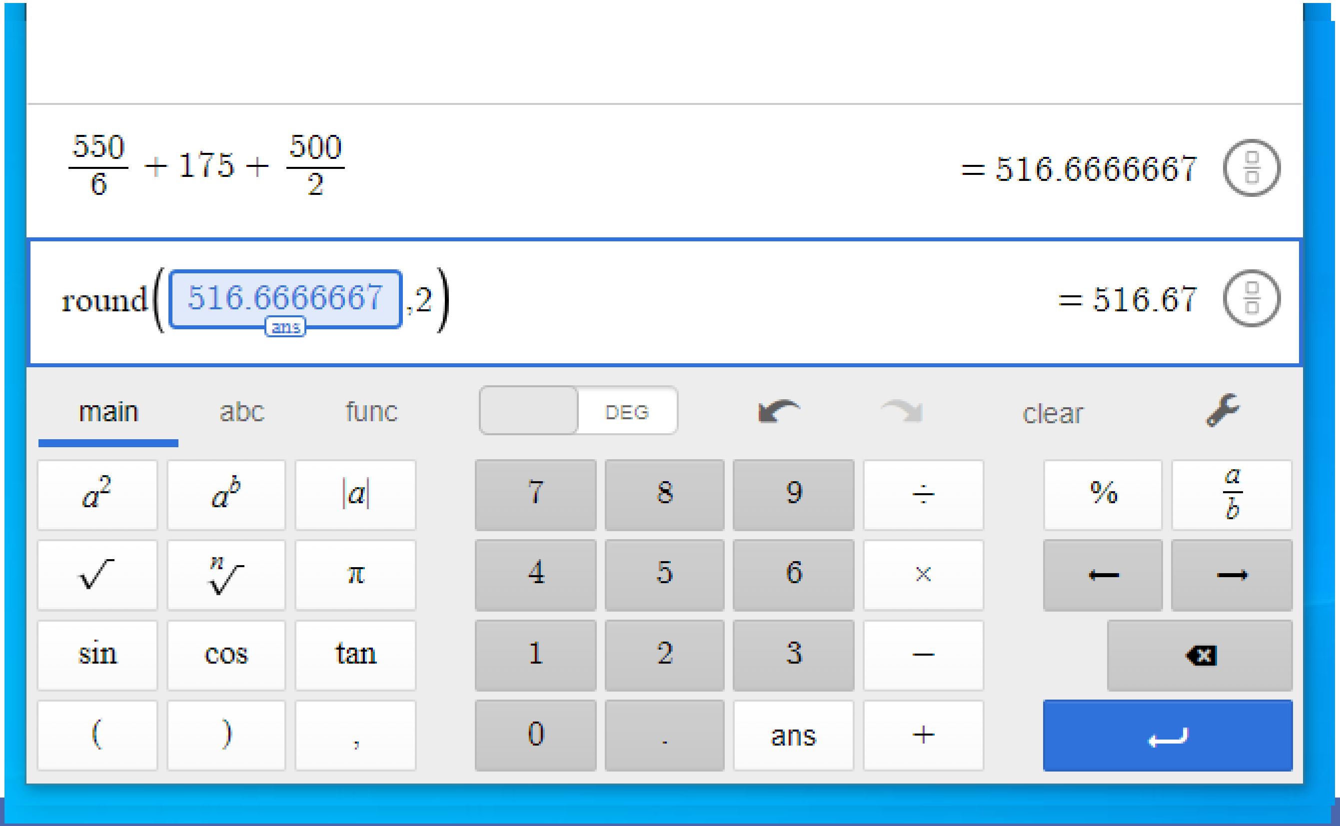
Try This Problem in Pairs

Prorate the following expenses and find the corresponding monthly expense.

Lan pays a semiannual premium of $550 for automobile insurance, a monthly premium of $175 for health insurance, and an annual premium of $500 for life insurance.

The monthly expense is $$\square$.

(Round to the nearest cent as needed.)



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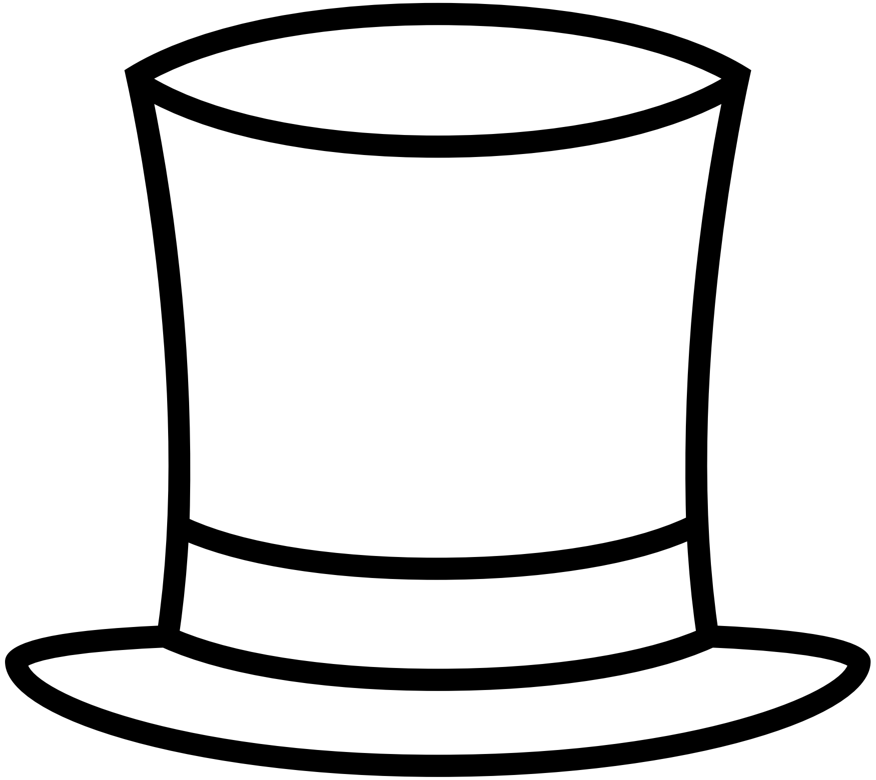


Unit 4C

Savings Plans and Investments

Example: Using the Savings Plan Formula (1 of 2)

Use the savings plan formula to calculate the balance after 6 months for an APR of 12% and monthly payments of $100.

Example: Using the Savings Plan Formula (2 of 2)

Solution

$$A = \text{PMT} \times \frac{\left[\left(1+\frac{\text{APR}}{n}\right)^{(nY)} - 1\right]}{\left(\frac{\text{APR}}{n}\right)} = \$100 \times \frac{\left[\left(1+\frac{0.12}{12}\right)^{(12\times 1/2)} - 1\right]}{\left(\frac{0.12}{12}\right)}$$

$$= \$100 \times \frac{\left[(1.01)^{6} - 1\right]}{0.01} = \$615.20$$

Example: A Comfortable Retirement (1 of 3)

You would like to retire 25 years from now and have a retirement fund from which you can draw an income of \$50,000 per year – forever! How can you do it? Assume a constant APR of 7%.

Solution

What balance do you need to earn \$50,000 from interest? Since we are assuming an APR of 7%, the \$50,000 must be 7% = 0.07 of the total balance.

$$\text{total balance} = \frac{\$50,000}{0.07} = \$714,286$$

Example: A Comfortable Retirement (2 of 3)

In other words, a balance of about $715,000 allows you to withdraw $50,000 per year without ever reducing the principle.

Let’s assume you will try to accumulate a balance of A = $715,000 by making regular monthly deposits into a savings plan. We have APR = 0.07, n = 12 (for monthly deposits) and Y = 25 years.

Example: A Comfortable Retirement (3 of 3)

$$\text{PMT} = \frac{A + \frac{\text{APR}}{n}}{\left[\left(1+\frac{\text{APR}}{n}\right)^{(nY)} - 1\right]} = \frac{\$715{,}000 + \frac{0.07}{12}}{\left[\left(1+\frac{0.07}{12}\right)^{(12\times 15)} - 1\right]}$$

$$= \frac{\$715{,}000 \times 0.0058333}{\left[(1.0058333)^{300} - 1\right]} = \$882.64$$

If you deposit \$883 per month over the next 25 years, you will receive your retirement goal.

Total Return

Consider an investment that grows from an original principal *P* to a later accumulated balance *A.*

The **total return** is the percentage change in the investment value:

$$\text{total return} = \frac{(A - P)}{P} \times 100\%$$

Annual Return

Consider an investment that grows from an original principal *P* to a later accumulated balance *A* in *Y* years.

The **annual return** is the annual percentage yield (APY) that would give the same overall growth.

$$\text{annual return} = \left(\frac{A}{P}\right)^{(1/Y)} - 1$$

Example: Mutual Fund Gain (1 of 2)

You invest \$3000 in the Clearwater mutual fund. Over 4 years, your investment grows in value to \$8400. What are your total and annual returns for the 4-year period?

Solution

$$\text{total return} = \frac{(A - P)}{P} \times 100\%$$

$$= \frac{(\$8400 - \$3000)}{\$3000} \times 100\% = 180\%$$

Example: Mutual Fund Gain (2 of 2)

$$\text{annual return} = \left(\frac{A}{P}\right)^{1/Y} - 1$$

$$= \left(\frac{\$8400}{\$3000}\right)^{1/4} - 1$$

$$= \sqrt[4]{2.8} - 1 \approx 0.294$$

$$= 29.4\%$$

Types of Investments (1 of 3)

Cash investments generally earn interest and include the following:

- Money you deposit into bank accounts
- Certificates of deposit (CD)
- U.S. Treasury bills

Types of Investments (2 of 3)

A **bond** (or debt) represents a promise of future cash.

- Buy a bond by paying some principal amount to the issuing government or corporation.
- The issuer pays you simple interest (as opposed to compound interest).
- The issuer promises to pay back your initial investment plus interest at some later date.

Types of Investments (3 of 3)

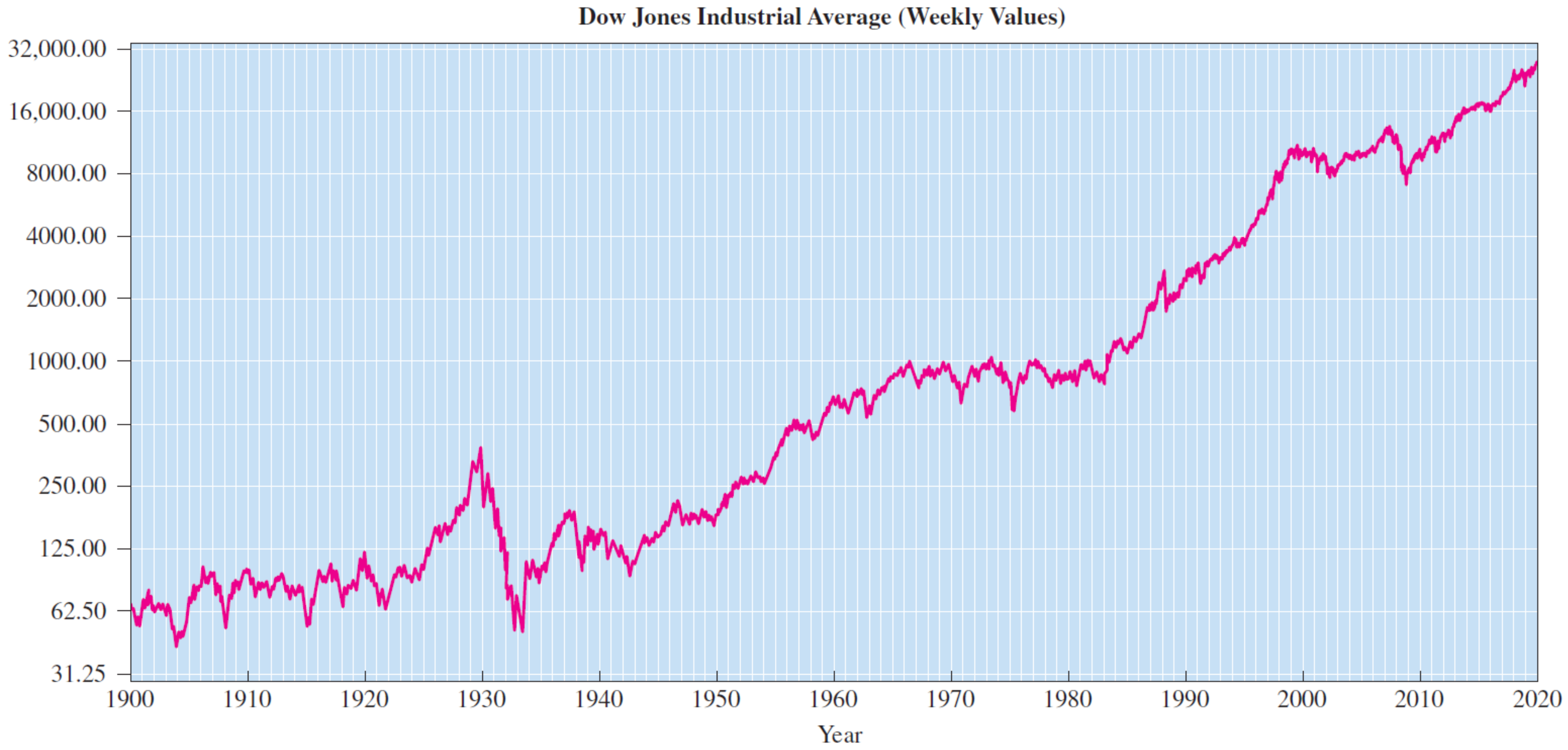
Stock (or equity) gives you a share of ownership in a company.

- Invest some principal amount to purchase the stock.
- The only way to get your money out is to sell the stock.
- Stock prices change with time, so the sale may give you either a gain or a loss on your original investment.

Investment Considerations

- **Liquidity:** How difficult is it to take out your money?
- **Risk:** Is your investment principal at risk?
- **Return:** How much return (total or annual) can you expect on your investment?

Stock Market Trends



The Dow Jones Industrial Average (DJIA) reflects the average prices of the stocks of 30 large companies.

Financial Data—Stocks

In general, there are two ways to make money on **stocks**:

1. Sell a stock for more than you paid for it, in which case you have a capital gain on the sale of the stock.
2. Make money while you own the stock if the corporation distributes part or all of its profits to stockholders as dividends.

Example: Understanding a Stock Quote (2 of 5)

a. What is the symbol for Microsoft stock?

b. What was the price per share at the start of the day?

c. Based on the current price, what is the total value of the shares that have been traded so far today?

d. What fraction of all Microsoft shares have been traded so far today?

e. Suppose you own 100 shares of Microsoft. Based on the current price and dividend yield, what total dividend should you expect to receive this year?

Example: Understanding a Stock Quote (3 of 5)

Solution

a. As shown at the top of the quote, Microsoft's stock symbol is MSFT.

b. The "Open" value is the price at the start of the day, which was $243.79.

Example: Understanding a Stock Quote (4 of 5)

c. The volume shows that 25,263,000 shares of Microsoft stock were traded today. At the current price of \$240.97 per share, the value of these shares is 25,263,000 shares $\times$ \$240.97/share $\approx$ \$6,088,000,000

So the total value of shares traded today is about \$6.1 billion.

Example: Understanding a Stock Quote (5 of 5)

d. We divide the 25,263,000 shares traded today by the total number of shares outstanding, which is quoted as 7542 million to find that about 0.0033, or 0.33%, of all shares have traded today.

e. At the current price, your 100 shares are worth $100 \times \$240.97 = \$24{,}097$. The dividend yield is 0.93%, so at that rate you would earn $\$24{,}097 \times 0.0093 = \$224.10$ in dividend payments this year.

Financial Data—Bonds

Bonds are issued with three main characteristics:

1. The face value (or par value) is the price you must pay the issuer to buy the bond.
2. The coupon rate of the bond is the *simple interest rate* that the issuer promises to pay.
3. The maturity date is the date on which the issuer promises to repay the face value of the bond.

$$\text{current yield} = \frac{\text{annual interest payment}}{\text{current price of bond}}$$

Example: Bond Interest

The closing price of a U.S. Treasury bond with a face value of $1000 is quoted as 105.97 points, for a current yield of 3.7%. If you buy this bond, how much annual interest will you receive?

Solution

$$105.97\% \times \$1000 = \$1059.70$$

$$\text{current yield} = \frac{\text{annual interest}}{\text{current price}}$$

$$\text{annual interest} = \text{current yield} \times \text{current price}$$

$$\text{annual interest} = 0.037 \times \$1059.70 = \$39.21$$

Financial Data—Mutual Funds

When comparing **mutual funds**, the most important factors are the following:

1. The fees charged for investing (not shown on most mutual fund tables)
2. How well the the funds perform

Note: Past performance is no guarantee of future results.

Example: Understanding a Mutual Fund Quote

Based on the Vanguard 500 mutual fund quote shown on the previous slide, how many shares will you be able to buy if you decide to invest $3000 in this fund today?

Solution

To find the number of shares you can buy, divide your investment of $3000 by the current share price, which is NAV of $361.25:

$$\frac{\$3000}{\$361.25} \approx 8.3$$

Your $3000 investment buys 8.3 shares in the fund.

Compute the total cost per year of the following pair of expenses. Then answer the question that follows.

Suppose you spend $22 every week on coffee and $136 per month on food. Over the course of a year, do you spend more on coffee or food?

Find the monthly interest payment in the situation below.

Vic bought a new plasma TV for $2300. He made a down payment of $100 and then financed the balance through the store. Unfortunately, he was unable to make the first monthly payment and now pays 5% interest per month on the balance (while he watches his TV).

For the following situation, find the average monthly expense that you would use in budgeting for the given expense. Note: Annual means once a year, and semiannual means twice a year.

Lan pays a semiannual premium of $700 for automobile insurance, a monthly premium of $160 for health insurance, and an annual premium of $500 for life insurance.

The expenses and income of an individual are given in table form to the right. Find the net monthly cash flow (it may be negative or positive). Assume that amounts shown for salaries and wages are after taxes and that 1 month = 4 weeks.

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	Health insurance: $450/month
	Car insurance: $500 twice a year
	Savings plan: $25/month
	Donations: $660/year
	Miscellaneous: $500/month

The net monthly cash flow is $☐.

(Round to the nearest dollar as needed.)

Use the compound interest formula to compute the balance in the following account after the stated period of time, assuming interest is compounded annually.

$14,000 invested at an APR of 4.7% for 24 years.

Find the savings plan balance after 3 years with an APR of 7% and monthly payments of $100.

Evaluate or simplify the following expression.

$2^4 + 4^2$

How much must be deposited today into the following account in order to have $55,000 in 6 years for a down payment on a house? Assume no additional deposits are made.

An account with annual compounding and an APR of 4%

Compute the total and annual return on the following investment.

Five years after paying $2100 for shares in a startup company, you sell the shares for $1400 (at a loss).

Company XYZ closed at $51.62 per share with a P/E ratio of 17.22. Answer the following questions.

a. How much were earnings per share?
b. Does the stock seem overpriced, underpriced, or about right given that the historical P/E ratio is 12-14?

QUESTIONS?

Compound Interest Formula
(for Interest Paid Once a Year)

$$A = P \times (1 + \mathrm{APR})^Y$$

A	=	accumulated balance after Y years
P	=	starting principal
APR	=	annual percentage rate (as a decimal)
Y	=	number of years

Compound Interest Formula for Interest Paid *n* Times per Year

$$A = P\left(1+\frac{\text{APR}}{n}\right)^{(nY)}$$

A	=	accumulated balance after Y years
P	=	starting principal
APR	=	annual percentage rate (as a decimal)
n	=	number of compounding periods per year
Y	=	number of years

Definition

- The **annual percentage yield (APY)** – also called the *effective yield* or simply the *yield* – is the actual percentage by which a balance increases in one year. It is equal to the APR if interest is compounded annually. It is greater than the APR if interest is compounded more than once a year.

APR vs. APY

APR = annual percentage rate

APY = annual percentage yield

APY = APR if interest is compounded annually

APY > APR if interest is compounded more than once a year

Compound Interest Formula for Continuous Compounding

$$A = P \times e^{(\mathrm{APR} \times Y)}$$

A	=	accumulated balance after Y years
P	=	starting principal
APR	=	annual percentage rate (as a decimal)
Y	=	number of years
e	=	a special irrational number with a value of $e \approx 2.71828$

Example: Continuous Compounding

You deposit \$100 in an account with an APR of 8% and continuous compounding. How much will you have after 10 years?

Solution

We have P = \$100, APR = 0.08, and Y = 10 years of continuous compounding.

The accumulated balance after 10 years is

$$A = P \times e^{\left(APR \text{ x } Y\right)} = \$100 \times e^{(0.08 \times 10)}$$

$$= \$100 \times e^{0.8} = \$222.55$$

Evaluate or simplify the following expression.

$$2^4 + 4^2$$

$$16 + 16$$

$$32$$

Evaluate the following expression.

$$64^{\frac{1}{2}} \qquad = 8$$

Compute the total cost per year of the following pair of expenses. Then answer the question that follows.

Suppose you spend $22 every week on coffee and $136 per month on food. Over the course of a year, do you spend more on coffee or food?

$52\ weeks\ per\ year$

$12\ months\ per\ year$

$\$22 \times 52$

$\$136 \times 12$

$\$1144$

$\$1632$

$$\$1632 - \$1144$$

$$Spend\ \$488\ more\ on\ food\ than\ coffee$$

Exodus 22:25

25 “If you lend money to one of my people among you who is needy, do not treat it like a business deal; charge no interest.

Use the $\boxed{\text{compound interest}}$ formula to determine the accumulated balance after the stated period.

$8000 invested at an APR of 8% for 5 years.

Annual

$$8000(1+8\%)^5 = \boxed{8000(1.08)^5}$$

$8000(1.08)^5$	$= 11754.62461$
round(11754.62461 ans ,2)	$= 11754.62$

Use the appropriate compound interest formula to compute the balance in the account after the stated period of time

$3,000 is invested for 18 years with an APR of 6% and $\boxed{\text{monthly compounding.}}$

Annual

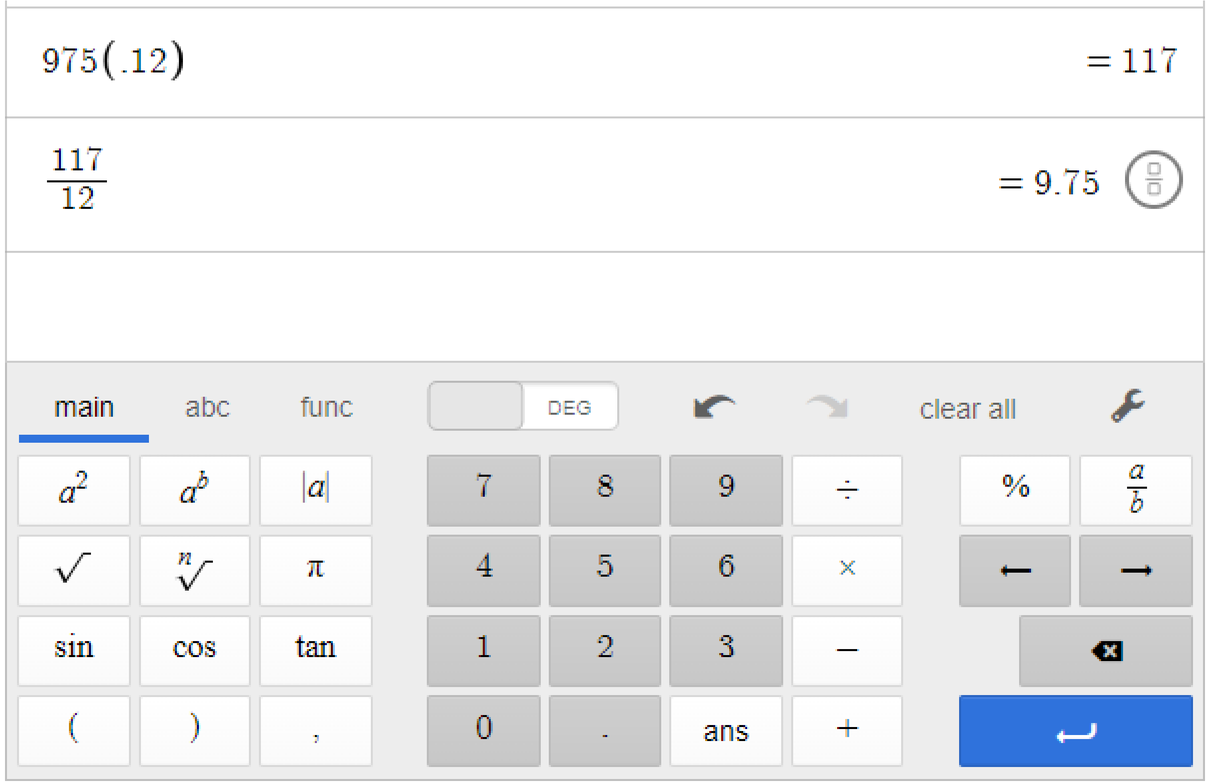
$$(3000)\left(1+\frac{6\%}{12}\right)^{18\cdot 12} = 3000(1.005)^{216} =$$

Try This Problem in Pairs

Find the monthly interest payment in the situation described below. Assume that the monthly interest rate is 1/12 of the annual interest rate.

You maintain an average balance of $975 on your credit card, which carries a 12% annual interest rate.

The monthly interest payment is $ ☐.
(Type an integer or a decimal.)



Savings Plan Formula (Regular Payments)

$$A = \text{PMT} \times \frac{\left[\left(1 + \frac{\text{APR}}{n} \right)^{(nY)} - 1 \right]}{\left(\frac{\text{APR}}{n} \right)}$$

A = accumulated savings plan balance

PMT = regular payment (deposit) amount

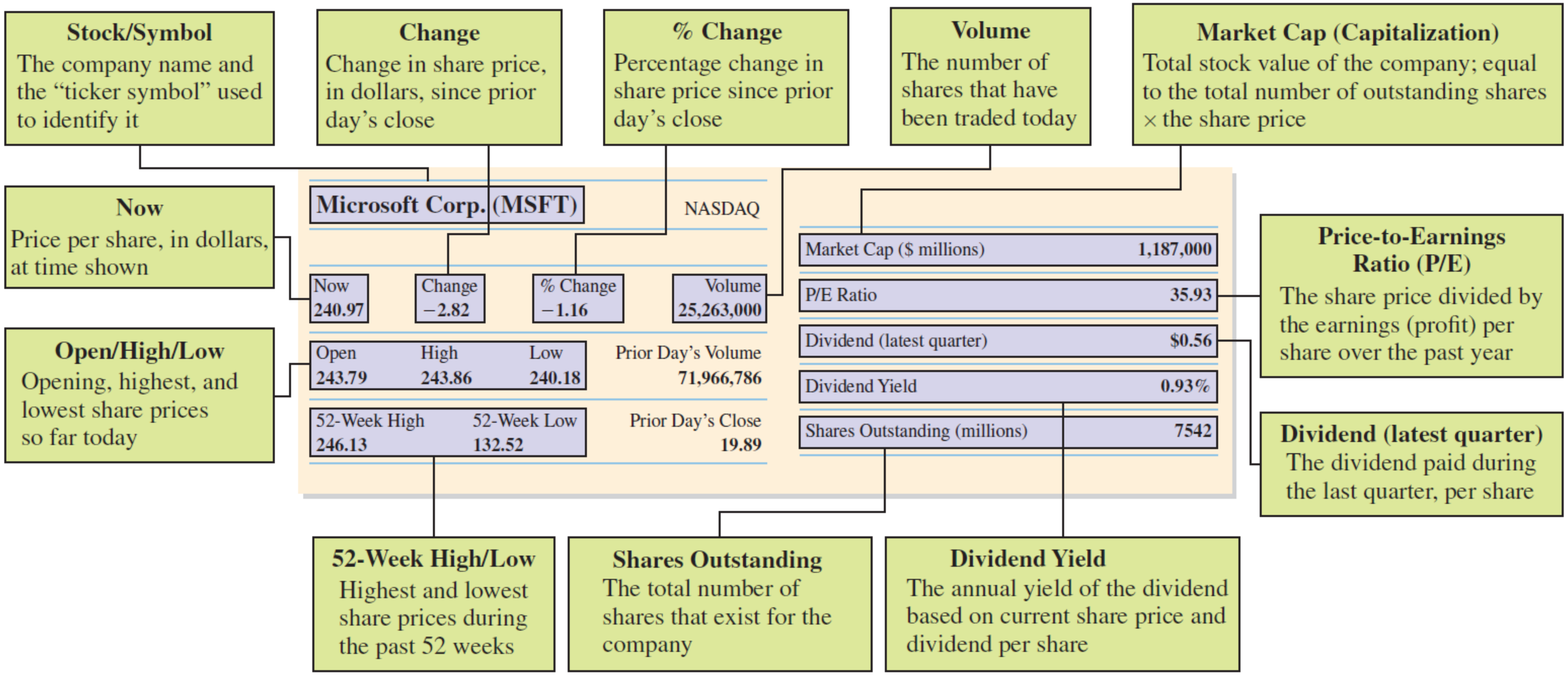
APR = annual percentage rate (as a decimal)

n = number of payment periods per year

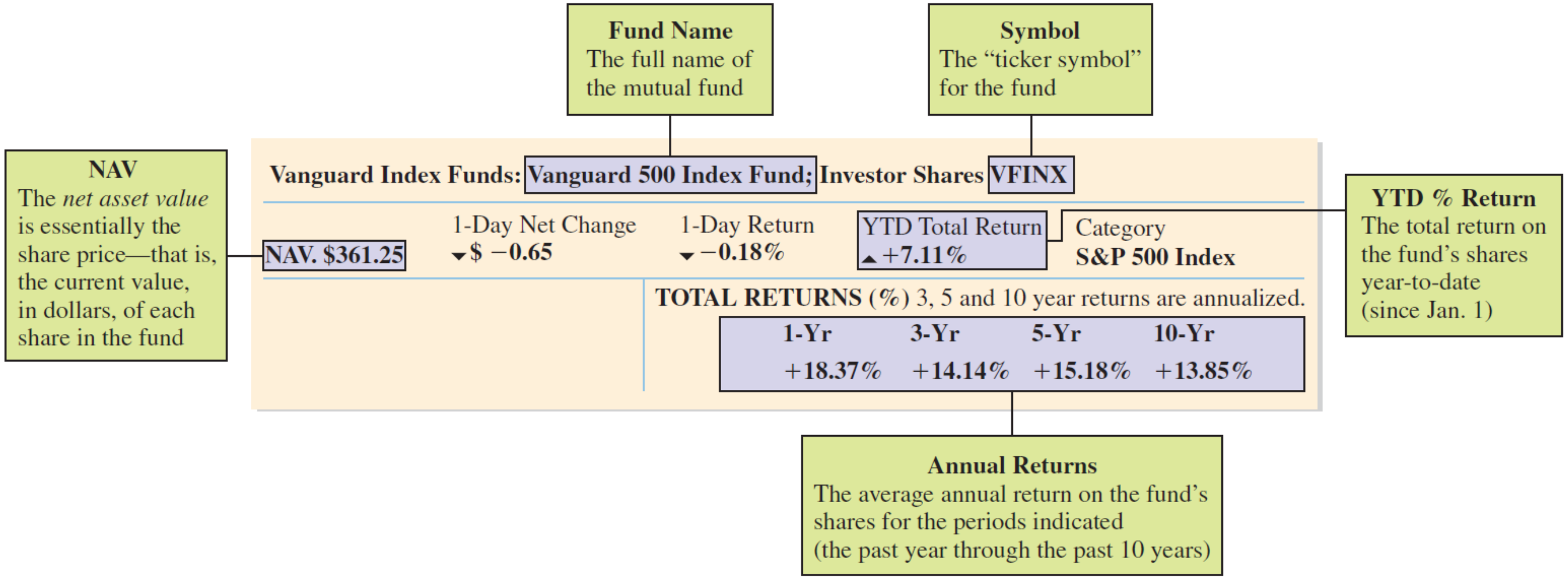
Y = number of years

Example: Understanding a Stock Quote (1 of 5)

Answer the following questions by assuming that the figure shows an actual Microsoft stock quote that you found online today.



Mutual Fund Quotations



Use the compound interest formula to determine the accumulated balance after the stated period.

$8000 invested at an APR of 8% for 5 years.

	A	B
1	How much to save now	=PV(
2	How much will I have in the future	=FV(
3	How much to save/pay monthly	=PMT(

($11,754.62)

=FV(.08,5,0,8000

FV(rate, nper, pmt, [pv], [type])

Description

Returns the future value of an investment based on periodic, constant payments and a constant interest rate

Example

=FV(0.06/12, 10, -200, -500, 1)

rate	is the interest rate per period. For example, use 6%/4 for quarterly payments at 6% APR
nper	is the total number of payment periods in the investment
pmt	is the payment made each period; it cannot change over the life of the investment
[pv]	is the present value, or the lump-sum amount that a series of future payments is worth now. If omitted, Pv = 0
[type]	is a value representing the timing of payment: payment at the beginning of the period = 1; payment at the end of the period = 0 or omitted

Find the savings plan balance after 3 years with an APR of 7% and monthly payments of $100.

	A	B
1	How much to save now	=PV(
2	How much will I have in the future	=FV(
3	How much to save/pay monthly	=PMT(

=FV(0.07/12,3*12,100)

($3,993.01)

FV(rate, nper, pmt, [pv], [type])

Description

Returns the future value of an investment based on periodic, constant payments and a constant interest rate

Example

=FV(0.06/12, 10, -200, -500, 1)

rate	is the interest rate per period. For example, use 6%/4 for quarterly payments at 6% APR
nper	is the total number of payment periods in the investment
pmt	is the payment made each period; it cannot change over the life of the investment
[pv]	is the present value, or the lump-sum amount that a series of future payments is worth now. If omitted, Pv = 0
[type]	is a value representing the timing of payment: payment at the beginning of the period = 1; payment at the end of the period = 0 or omitted

Learn more about FV

Give feedback

How much must be deposited today into the following account in order to have $55,000 in 6 years for a down payment on a house? Assume no additional deposits are made.

An account with annual compounding and an APR of 4%

	A	B
1	How much to save now	=PV(
2	How much will I have in the future	=FV(
3	How much to save/pay monthly	=PMT(

($43,467.30)

=PV(0.04/1,6*1,0,55000

PV(rate, nper, pmt, [fv], [type])

Description

Returns the present value of an investment: the total amount that a series of future payments is worth now

Example

=PV(0.08/12, 12*20, 500, , 0)

rate	is the interest rate per period. For example, use 6%/4 for quarterly payments at 6% APR
nper	is the total number of payment periods in an investment
pmt	is the payment made each period and cannot change over the life of the investment
[fv]	is the future value, or a cash balance you want to attain after the last payment is made
[type]	is a logical value: payment at the beginning of the period = 1; payment at the end of the period = 0 or omitted

Learn more about PV

Give feedback

Use the compound interest formula for compounding more than once a year to determine the accumulated balance after the stated period.

$12,000 deposit at an APR of 4% with semiannual compounding for 5 years

	A	B
1	How much to save now	=PV(
2	How much will I have in the future	=FV(
3	How much to save/pay monthly	=PMT(

($14,627.93)

=FV(.04/2,5*2,0,12000)

FV(rate, nper, pmt, [pv], [type])

Description

Returns the future value of an investment based on periodic, constant payments and a constant interest rate

Example

=FV(0.06/12, 10, -200, -500, 1)

rate	is the interest rate per period. For example, use 6%/4 for quarterly payments at 6% APR
nper	is the total number of payment periods in the investment
pmt	is the payment made each period; it cannot change over the life of the investment
[pv]	is the present value, or the lump-sum amount that a series of future payments is worth now. If omitted, Pv = 0
[type]	is a value representing the timing of payment: payment at the beginning of the period = 1; payment at the end of the period = 0 or omitted

Learn more about FV

Give feedback

Compute the total and annual returns on the following investment.

Seventeen years after purchasing shares in a mutual fund for \$5900, the shares are sold for \$11,100.

$$\text{total return} = \frac{(11100 - 5900)}{5900} \times 100\% \boxed{= 88.1\%}$$

$$\text{annual return} = \left(\frac{11100}{5900}\right)^{(1/17)} - 1 \quad = .038 \times 100\% \boxed{= 3.8\%}$$

$$\boxed{\text{total return} = \frac{(A - P)}{P} \times 100\%}$$

$$\boxed{\text{annual return} = \left(\frac{A}{P}\right)^{(1/Y)} - 1}$$

Company XYZ closed at $99.86 per share with a P/E ratio of 10.91. Answer the following questions.

a. How much were earnings per share?
b. Does the stock seem overpriced, underpriced, or about right given that the historical P/E ratio is 12-14?

$$\frac{P}{E} = 10.91 \longrightarrow \frac{\$99.86}{E} = 10.91 \longrightarrow \frac{\$99.86}{10.91} = E$$

$$\boxed{E = \$9.15}$$

$$\boxed{Underpriced}$$

How long will it take money to triple at an APR of 8.5% compounded annually?

$$\boxed{Principal + 200\%\ increase = triple}$$

$$\boxed{\frac{\log(1 + total\ return)}{\log(1 + annual\ return)} = Years}$$

$$\frac{\log(1 + 200\%)}{\log(1 + 8.5\%)} = Years$$

$$\frac{\log(1 + 2)}{\log(1 + .085)} = \frac{\log(3)}{\log(1.085)} = 13.47$$

14 years

(Round up to the nearest year.)

$1000 is deposited in an account that pays an APR of 6.5% compounded annually.

How long will it take for the balance to reach $160,000?

	A	B
1	How much to save now	=PV(
2	How much will I have in the future	=FV(
3	How much to save/pay monthly	=PMT(

$$\text{total return} = \frac{(160000 - 1000)}{1000} \times 100\% \boxed{= 15900\%}$$

$$\frac{\log(1+159)}{\log(1+.065)} = \frac{\log(160)}{\log(1.065)} = 80.59$$

81 years

(Round up to the nearest year.)

$$\boxed{\text{total return} = \frac{(A-P)}{P} \times 100\%}$$

$$\boxed{\frac{log(1 + total\ return)}{log(1 + annual\ return)} = Years}$$

Compute the total and annual returns on the following investment.

Seventeen years after purchasing shares in a mutual fund for \$5900, the shares are sold for \$11,100.

$$\frac{5200}{5900} = .881$$

88.1%

$$\begin{array}{r} \$11,100 \\ -\$5,900 \\ \hline \$5,200 \end{array}$$

$$\boxed{3.8\%}$$

$$\sqrt[17]{1 + 88.1\%} \rightarrow \sqrt[17]{1.881}$$

$$(1.881)^{\frac{1}{17}}$$

Company XYZ closed at $99.86 per share with a P/E ratio of 10.91. Answer the following questions.

a. How much were earnings per share?
b. Does the stock seem overpriced, underpriced, or about right given that the historical P/E ratio is 12-14?

$$\frac{99.86}{E} = 10.91$$

$$E = \frac{99.86}{10.91} = \text{earnings per shar}$$

Try This Problem in Pairs

Consider the following loan. Complete parts (a)-(c) below.

An individual borrowed $84,000 at an APR of 3%, which will be paid off with monthly payments of $466 for 20 years.

---

a. Identify the amount borrowed, the annual interest rate, the number of payments per year, the loan term, and the payment amount.

The amount borrowed is $ the annual interest rate is %, the number of payments per year is , the loan term is years, and the payment amount is $.

b. How many total payments does the loan require? What is the total amount paid over the full term of the loan?

There are payments toward the loan and the total amount paid is $.

c. Of the total amount paid, what percentage is paid toward the principal and what percentage is paid for interest?

The percentage paid toward the principal is % and the percentage paid for interest is %.
(Round to the nearest tenth as needed.)

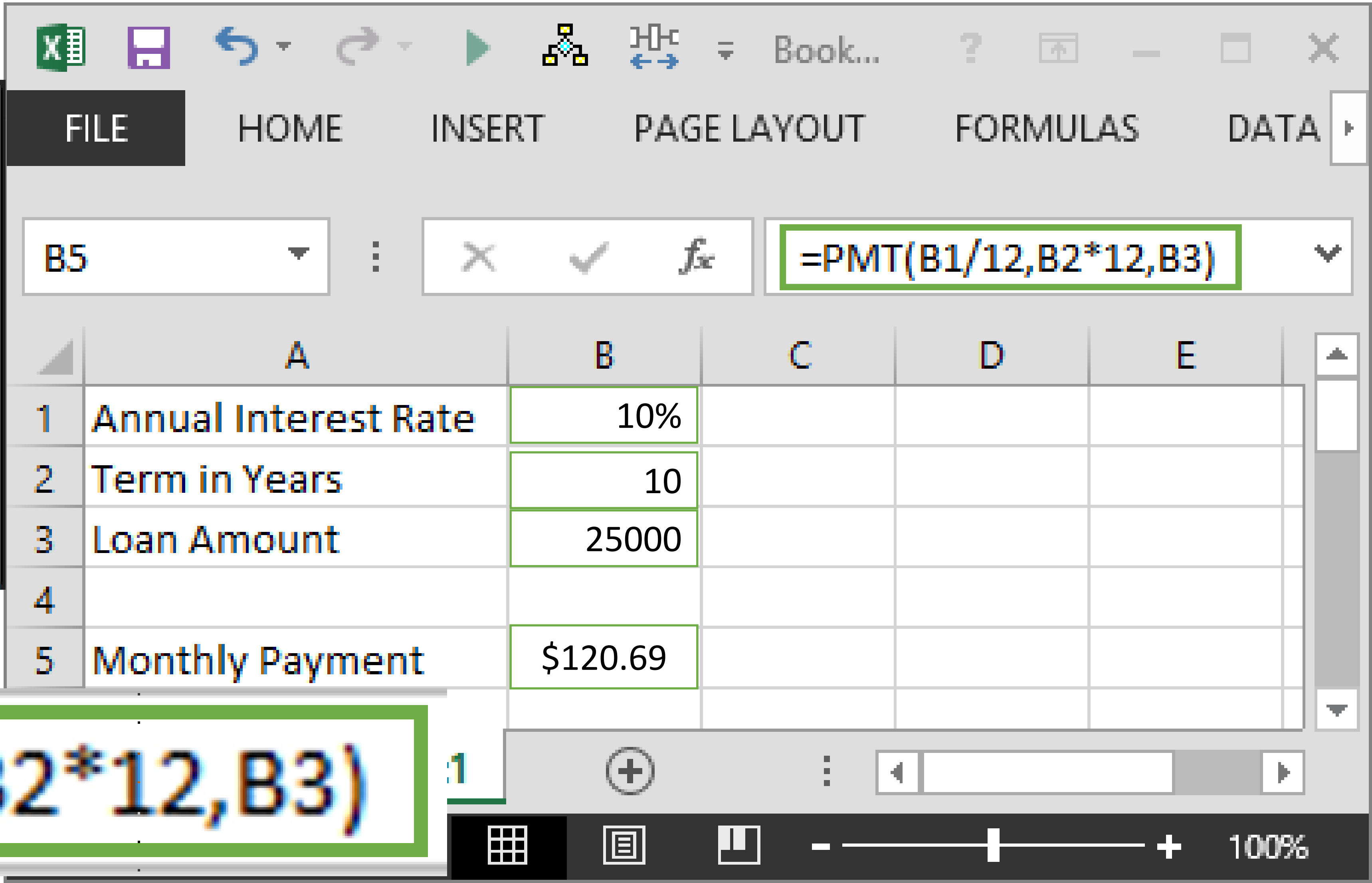
Consider a student loan of $15,000 at a fixed APR of 9% for 30 years.

a. Calculate the monthly payment.
b. Determine the total amount paid over the term of the loan.
c. Of the total amount paid, what percentage is paid toward the principal and what percentage is paid for interest.

Formula of the Day

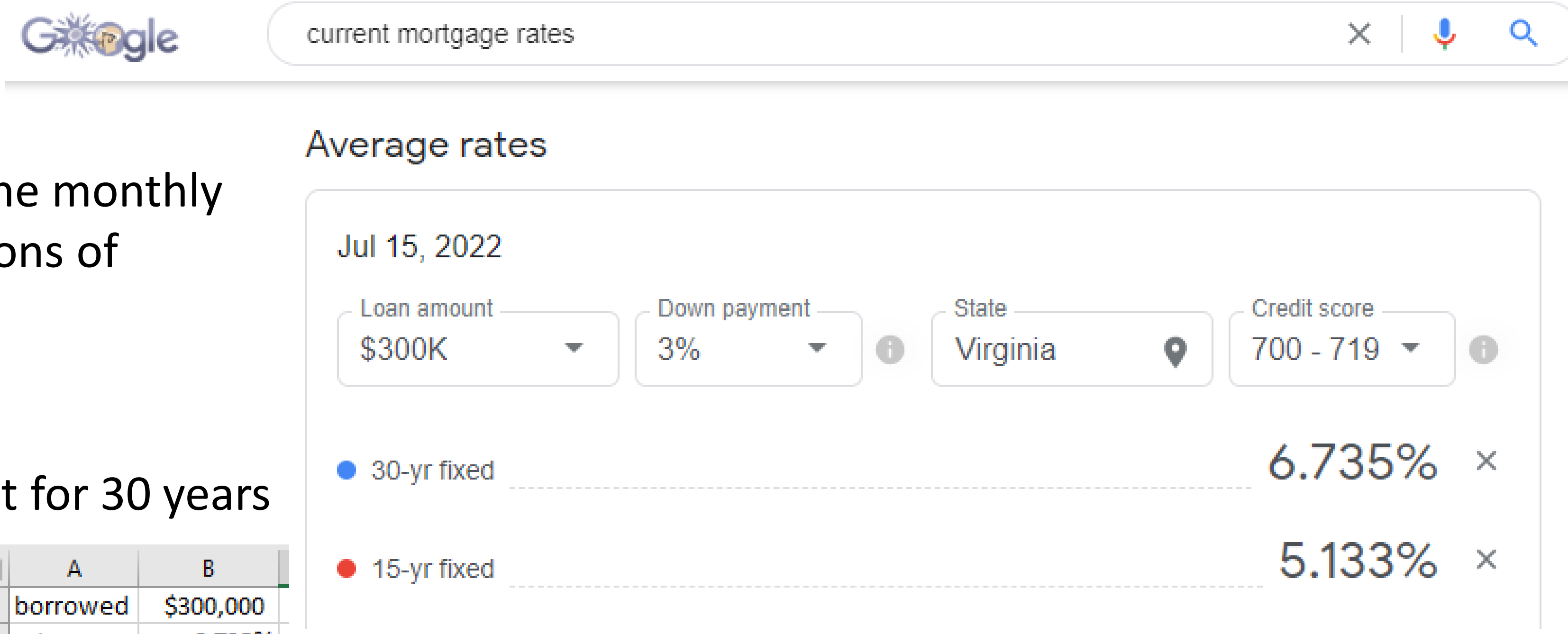
$$A = P\frac{r(1+r)^n}{(1+r)^n - 1}$$

A = payment Amount per period
P = initial principle (loan amount)
r = interest rate per period
n = total # of payments (periods)



=PMT(B1/12,B2*12,B3)

Try This Problem in Pairs

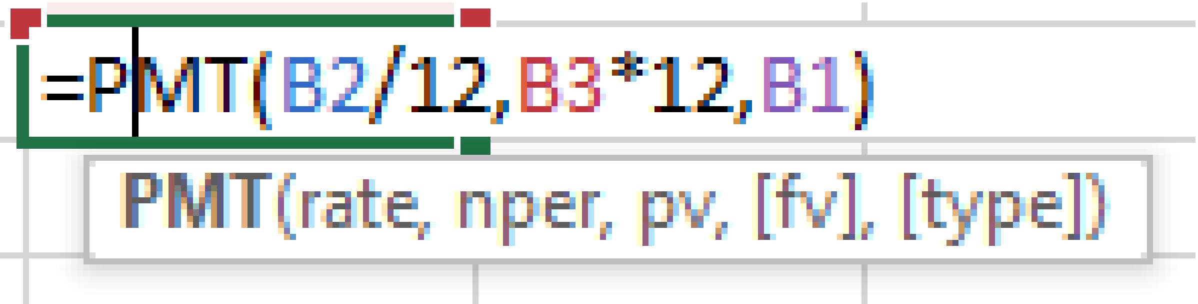


Use Excel PMT function to calculate the monthly payment for the following two situations of financing a home:

Situation A:
$300,000 borrowed at 6.735% interest for 30 years

	A	B	C
1	borrowed	$300,000	
2	rate	6.735%	
3	years	30	
4	PMT	=PMT(B2/12,B3*12,B1)	
5			

	A	B
1	borrowed	$300,000
2	rate	6.735%
3	years	30
4	PMT	($1,942.80)

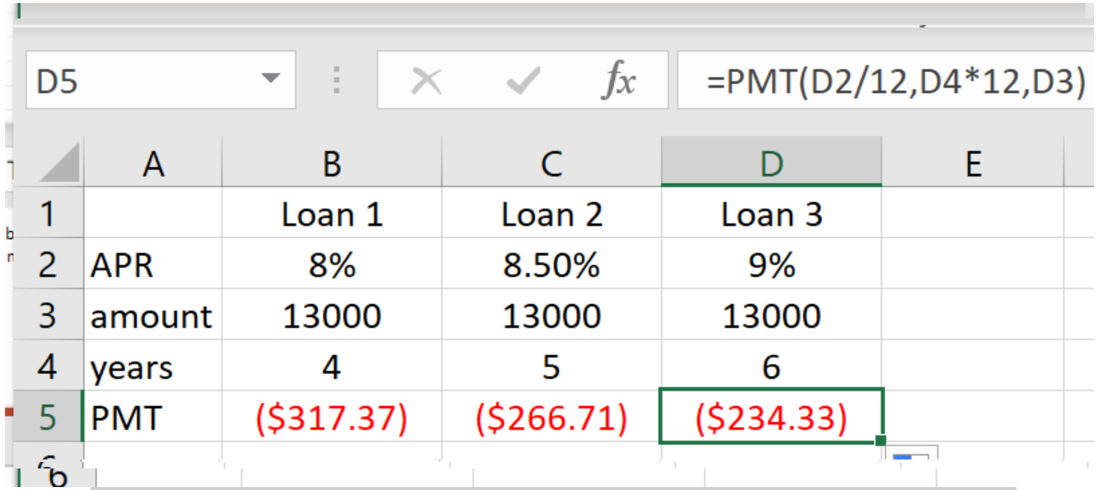


Situation B:
$300,000 borrowed at 5.133% interest for 15 years

	A	B
1	borrowed	$300,000
2	rate	5.133%
3	years	15
4	PMT	($2,393.22)

Comparing Loans Example

Someone needs to borrow $13,000 to buy a car and the person has determined that monthly payments of $250 are affordable. The bank offers a 4-year loan at 8% APR, a 5-year loan at 8.5%, or a 6-year loan at 9% APR. Which loan best meets the person's needs? Explain.



D5 =PMT(D2/12,D4*12,D3)

	A	B	C	D	E
1		Loan 1	Loan 2	Loan 3	
2	APR	8%	8.50%	9%	
3	amount	13000	13000	13000	
4	years	4	5	6	
5	PMT	($317.37)	($266.71)	($234.33)