Math 114 – Module 2

Mr. Andrew Volk

Spring 2025

Today’s Agenda

- Time Spent Reflection
- Multiplying Fractions
- Adding/subtracting Fractions
- Preview this week’s assignments
- Tophat
- Prayer
- Plan this week

What to do in lecture?

Show up

Take notes

Key formulas

New words

Hints and shortcuts

Answer questions

Make Progress

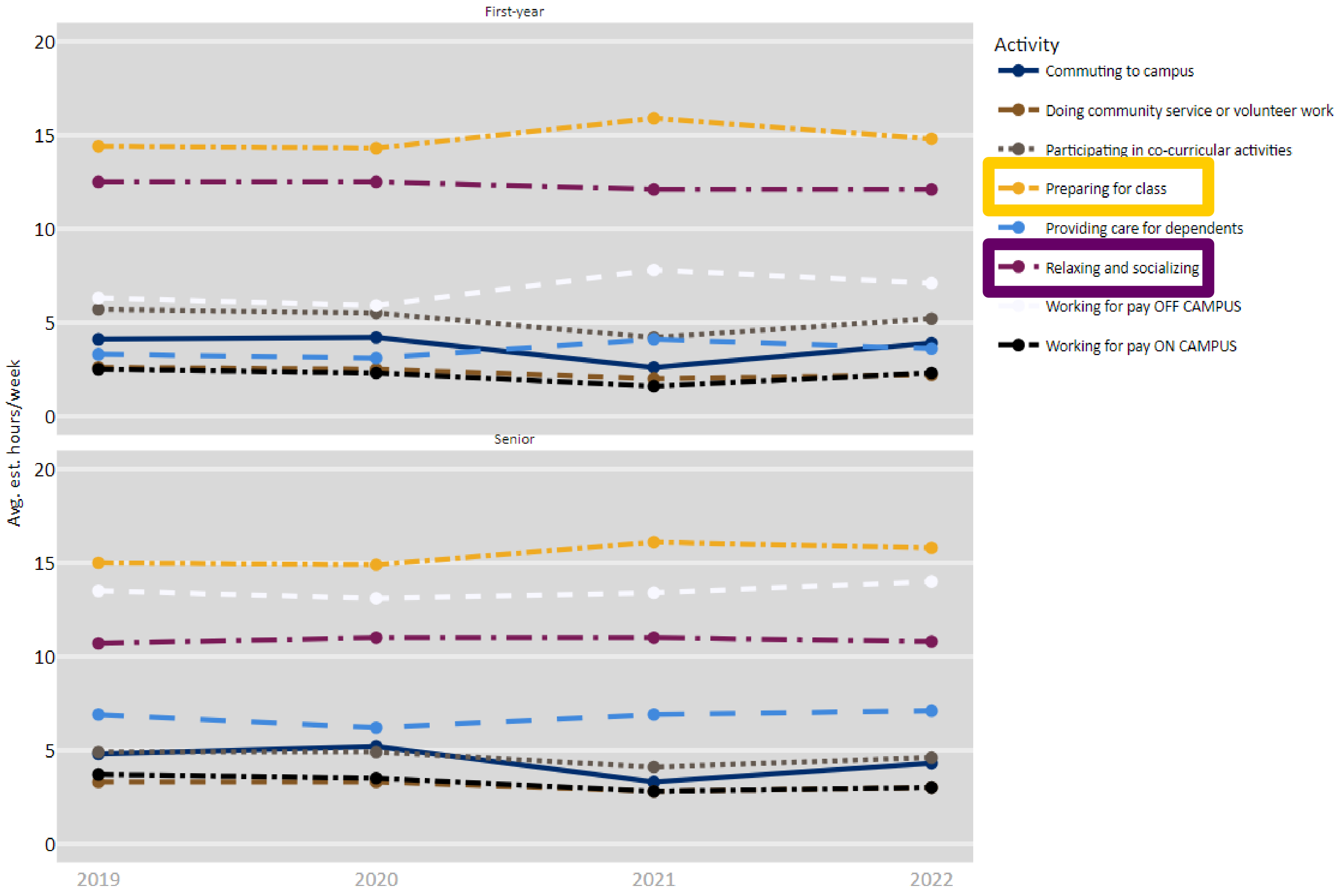
How are College Students Spending their Time?

Assign a number between 0-168 to each of the tasks.

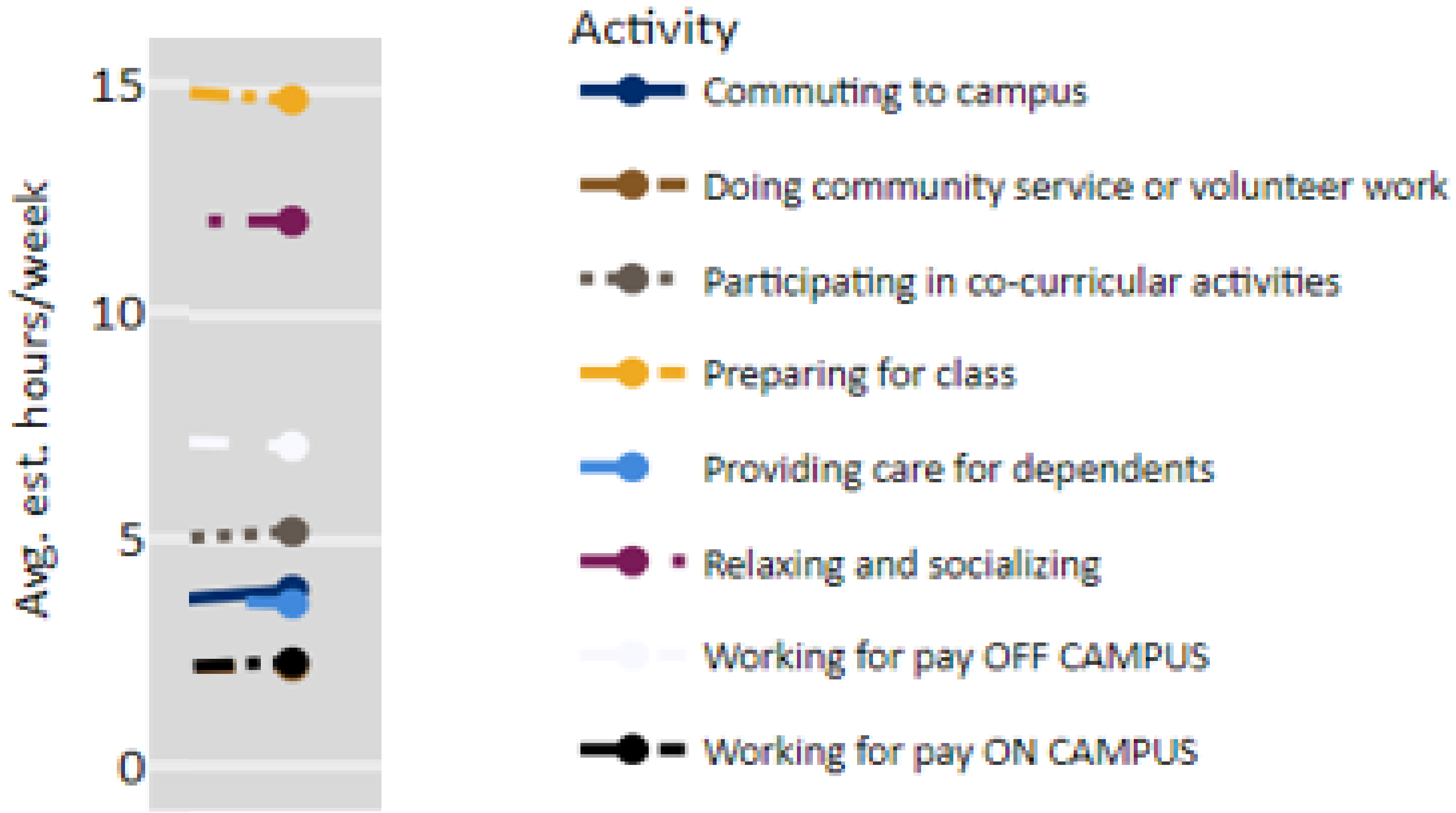
How many hours per week do you spend doing each task on average?

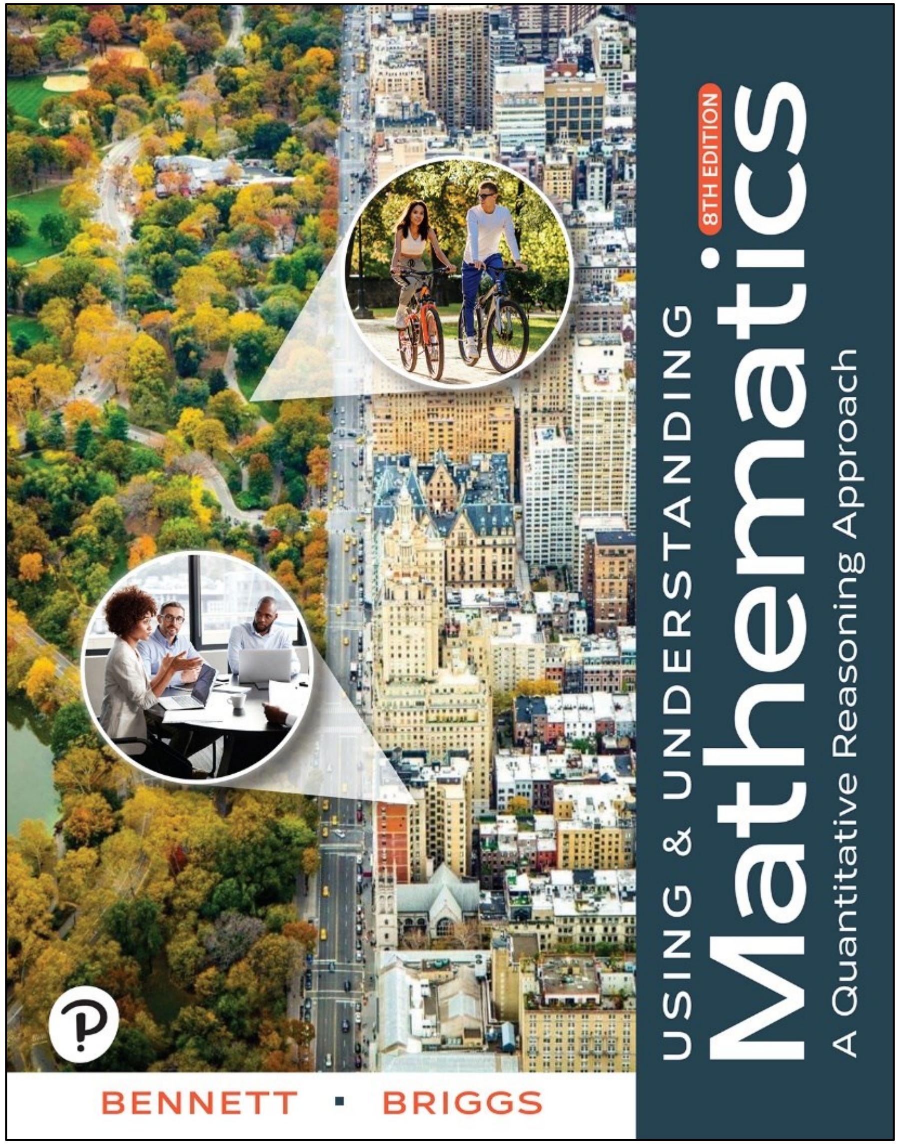
Activity

- A Commuting to campus
- B Doing community service or volunteer work
- C Participating in co-curricular activities
- D Preparing for class
- E Providing care for dependents
- F Relaxing and socializing
- G Working for pay OFF CAMPUS
- H Working for pay ON CAMPUS


First-year
Senior
Avg. est. hours/week
20
15
10
5
0
2019
2020
2021
2022
Activity
Commuting to campus
Doing community service or volunteer work
Participating in co-curricular activities
Preparing for class
Providing care for dependents
Relaxing and socializing
Working for pay OFF CAMPUS
Working for pay ON CAMPUS

How are First Year College Students Spending their Time?





This Week’s Reading

Chapter 2

(Sections A, B, & C)

Approaches to Problem Solving

Homework 2: 20 Questions

Due Sunday 11:59 pm

Questions 1-2	• Decide if a statement involving units makes sense.
Question 3	• Add and multiply fractions.
Questions 4-5	• Convert between fractions and decimals.
Questions 6 & 8	• Describe the steps to apply the Understand-Solve-Explain process.
Questions 7 & 9	• Solve applications involving units.
Questions 10-11	• Determine appropriate units and perform unit conversions.
Question 12	• Identify and define words, phrases, and specific processes involving unit analysis.
Question 13	• Decide if a statement involving units makes sense.
Question 14	• Solve applications involving unit analysis.
Question 15	• Solve applications involving conversions within the USCS system.
Questions 16-19	• Convert units.
Question 20	• Solve applications involving temperature conversions.

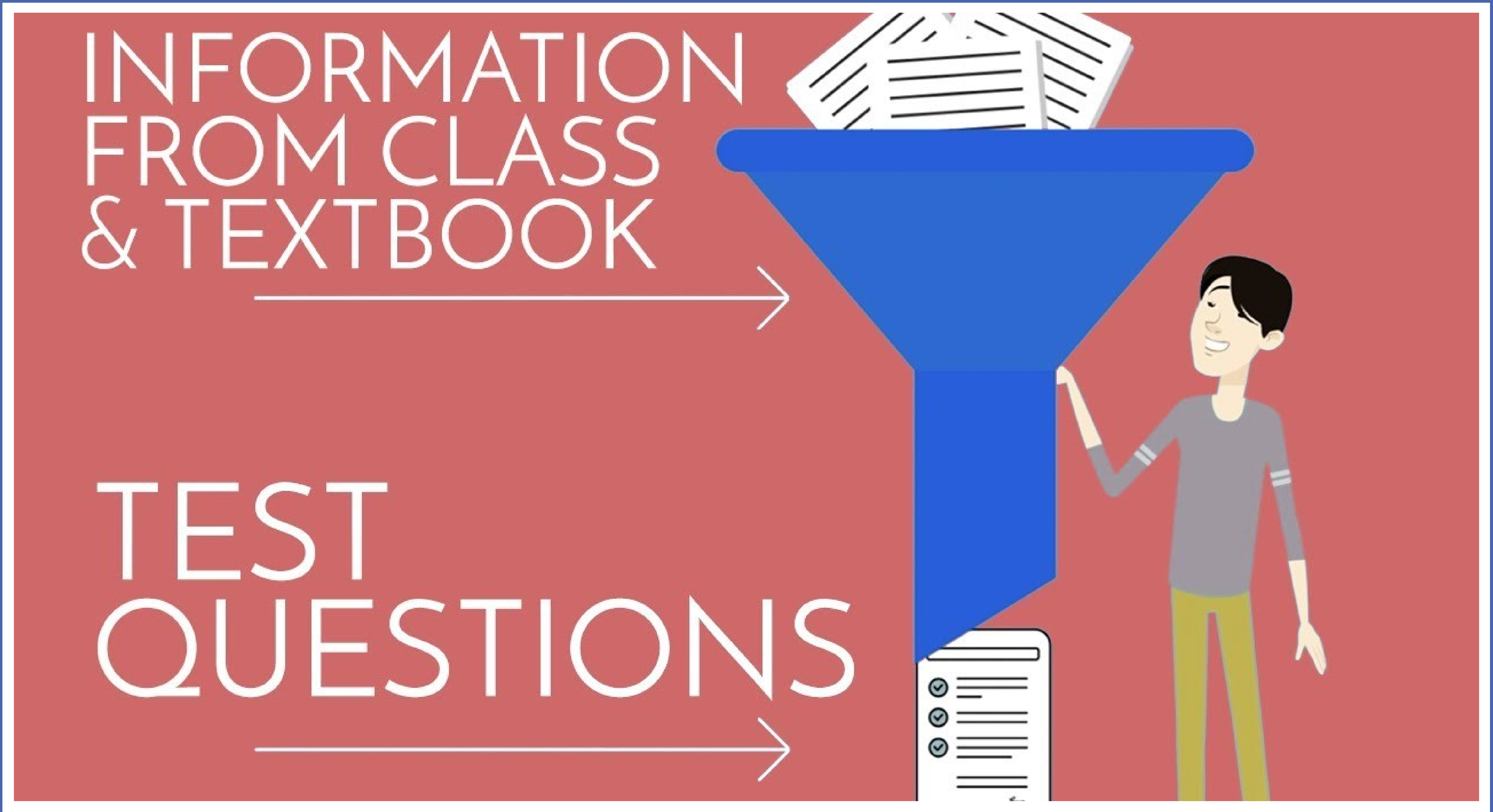
Quiz 2: 10 Questions

Due Monday
11:59 pm

Question 1	• Determine appropriate units and perform unit conversions.
Question 2	• Decide if a statement involving units makes sense.
Question 3	• Identify and define words, phrases, and specific processes involving units.
Question 4	• Add and multiply fractions.
Question 5	• Convert between fractions and decimals.
Question 6	• Solve applications involving units.
Question 8	• Solve applications involving conversions within the USCS system.
Question 9	• Convert units.
Question 10	• Solve applications involving temperature conversions.

Module 2: **Learning Outcomes**

- Add and Multiply Fractions
- Convert between fractions and decimals.
- Interpret units and perform unit conversions.
- Solve applications involving units.
- Solve applications involving temperature conversions.

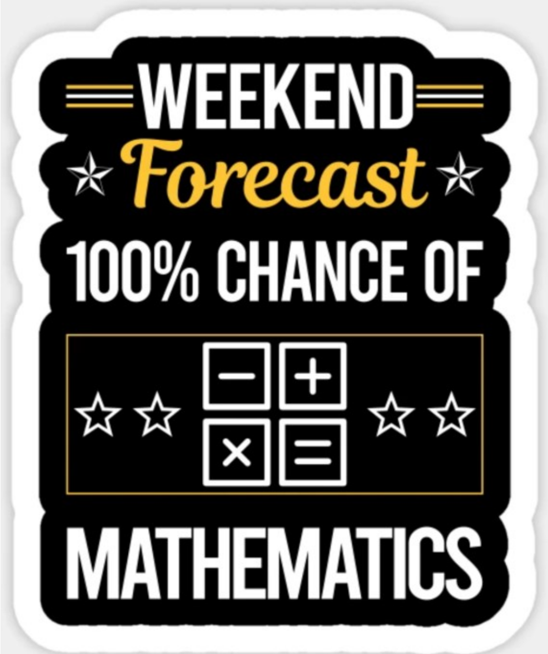

INFORMATION FROM CLASS & TEXTBOOK
TEST QUESTIONS

The Active Student

Start Assignments Early → Use Resources → Ask Questions → High Standards

Help me solve this

Ask my instructor



The Responsive Professor

Provide Clear Lectures → Provide Resources → Answer Questions → High Standards

Understand → Solve → Explain

- Think about what the problem asks you to do.
- Draw a picture or diagram to help make sense of the problem.
- Ask yourself what the solution should look like.
- Try to map a path (either mentally or in writing) that will lead you from your understanding of the problem to its solution.
- Continually revisit your understanding of the problem.

NOTES

Understand → **Solve** → Explain

- Obtain any needed information or data.
- For multi-step problems, be sure to keep an organized, neatly written record of your work.
- Double-check each step as you work to avoid carrying errors through to the end of your solution.
- Constantly reevaluate your plan as you work.

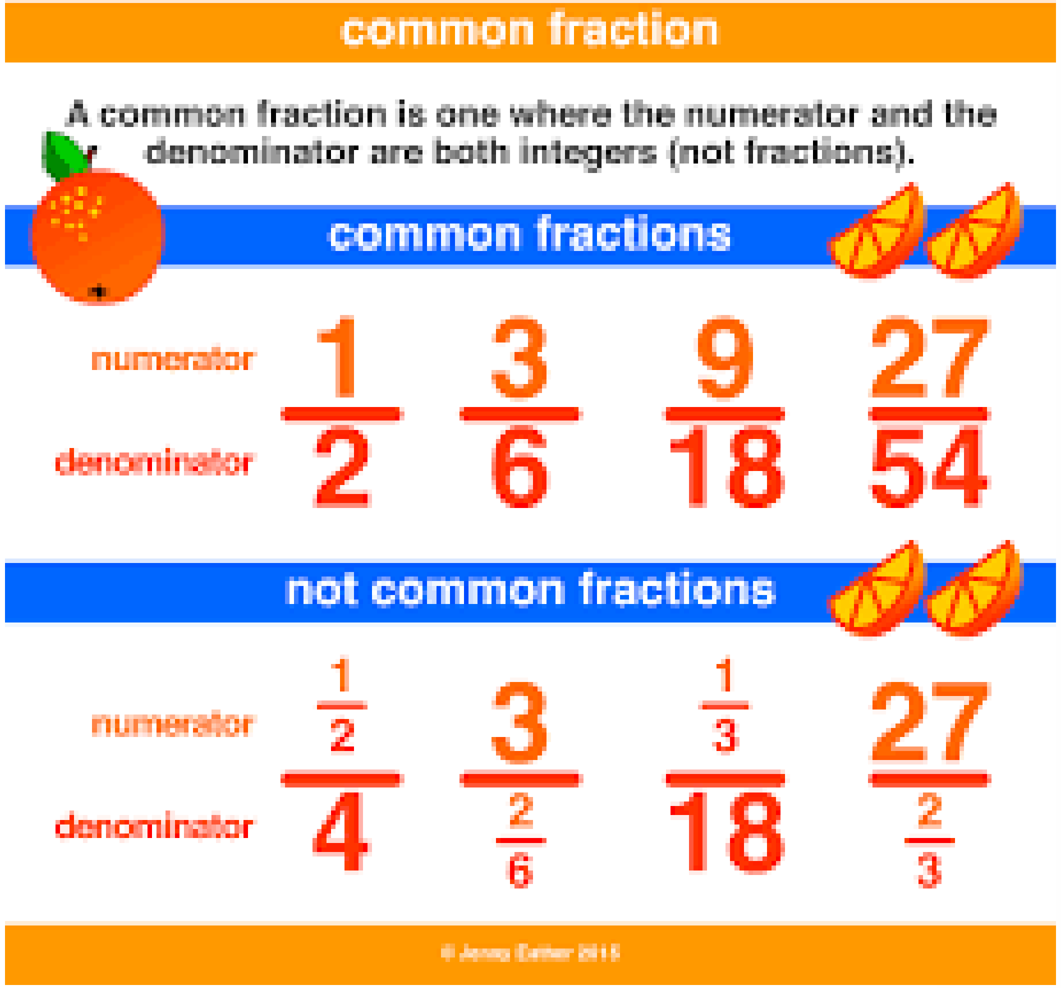
NOTES

Understand → Solve → Explain

- Be sure that your result makes sense.
- Recheck your calculations once more or, even better, find an independent way to check your result.
- Identify and understand any potential sources of uncertainty in your result. If you made assumptions, were they reasonable?
- Write your solution clearly and concisely, using complete sentences to make sure the context and meaning are clear.

NOTES

NOTES



Metric System Prefixes

Prefix	Symbol	Power	Factor	Example Unit
giga	G	10^9	1,000,000,000	
mega	M	10^6	1,000,000	
kilo	k	10^3	1,000	kilometer (km)
centi	c	10^{-2}	0.01	centimeter (cm)
milli	m	10^{-3}	0.001	millimeter (mm)
micro	μ	10^{-6}	0.000,001	micrometer (μm)
nano	n	10^{-9}	0.000,000,001	nanometer (nm)

NOTES

The **units** of a quantity describe what that quantity measures or counts.

Unit analysis is the process of working with units to help solve problems.

NOTES

Starting amount	Equal amounts		End Amount
24 inches	1 foot	=	feet
	12 inches		

24 ~~inches~~	1 foot	=	**2** feet
	12 ~~inches~~		

Dimensional Analysis

1. Write the 2 givens (starting, ending).
2. Fill the middle with conversion factors.
3. Make sure the units cancel out.
4. Solve the problem.

Example:

Convert 24 kilometers per minute to meters per second.

$$\frac{24\ \text{km}}{\text{min}} \qquad = \frac{\text{m}}{\text{s}}$$

$$\frac{24\ \cancel{\text{km}}}{\cancel{\text{min}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}} = \frac{400\ \text{m}}{\text{s}}$$

Convert 27 minutes to seconds.

There are $\boxed{}$ seconds in 27 minutes.
(Simplify your answer.)

$$\frac{27 \text{ } \cancel{\text{Minutes}}}{} \times \frac{60 \text{ Seconds}}{1 \text{ } \cancel{\text{Minutes}}} \rightarrow \frac{1620 \text{ Seconds}}{1} \rightarrow \boxed{1620 \text{ Seconds}}$$

Example 1

Use two or three unit fractions to convert.

12 weeks = ________ minutes

$$\text{12 }\cancel{\text{Weeks}} \times \frac{\text{7 }\cancel{\text{Days}}}{\text{1 }\cancel{\text{Week}}} \times \frac{\text{24 }\cancel{\text{Hours}}}{\text{1 }\cancel{\text{Day}}} \times \frac{\text{60 Minutes}}{\text{1 }\cancel{\text{Hour}}} \longrightarrow \boxed{\text{120960 Min.}}$$

Example 2

Convert the measurement to the units specified.

23 pounds to kilograms

1 in.	=	2.540 cm	1 cm	=	0.3937 in
1 ft	=	0.3048 m	1 m	=	3.28 ft
1 yd	=	0.9144 m	1 m	=	1.094 yd
1 mi	=	1.6093 km	1 km	=	0.6214 mi
1 lb	=	0.4536 kg	1 kg	=	2.205 lb

$$\frac{23\ \cancel{\text{pounds}}}{} \times \frac{0.4536\ \text{kg}}{1\ \cancel{\text{pound}}} \rightarrow \frac{10.4328\ \text{KG}}{1} \rightarrow \boxed{10.43\ \text{kilograms}}$$

Example 3

Use the sample currency exchange rates given in the table to answer the following question. State all of the conversion factors that you use.

Suppose that a new fuel-efficient European car travels an average of 23 kilometers on 1 liter of gasoline. If gasoline costs 1.25 euros per liter, how much will it cost to drive 200 kilometers in dollars?

Currency	Dollars per Foreign	Foreign per Dollar
British pound	1.414	0.7072
Canadian dollar	0.7834	1.277
European euro	1.256	0.7965
Japanese yen	0.01007	99.34
Mexican peso	0.06584	15.19

$$\frac{1\ \cancel{\text{Liter}}}{23\ \text{km}} \times \frac{1.25\ \cancel{\text{euro}}}{1\ \cancel{\text{Liter}}} \times \frac{1.256\ \text{USD}}{1\ \cancel{\text{euro}}} \rightarrow \frac{1.57\ \text{USD}}{23\ \text{km}} \times \frac{8.696}{8.696} \rightarrow \frac{\boxed{14.08\ \text{Dollars}}}{200\ \text{km}}$$

$200/23 = 8.696$

Example 4

Convert to Fahrenheit. Use $C = \frac{5}{9}(F - 32)$ or $F = \frac{9}{5}C + 32$, where F is the degrees in Fahrenheit and C is the degrees in Celsius.

$-50°$ C

$$F = \frac{9}{5}(-50) + 32 = -90 + 32$$

$$F = \boxed{-58°F}$$

Example 5

Convert the following temperatures from Fahrenheit to Celsius or vice versa.

$C = \frac{F - 32}{1.8}$, $F = 1.8C + 32$

a. $25^\circ F$ b. $40^\circ C$ c. $-20^\circ C$

$F = 1.8(-20) + 32$

$C = \frac{(25) - 32}{1.8}$

Example 6

MATH AUTOBIOGRAPHY

Math Autobiography

- ❑100 word minimum
- ❑Not graded for writing quality
- ❑5 points (bonus)

Math Autobiography Ideas

- Earliest Math Memory?
- Time you enjoyed math?
- Time you did not enjoy math?
- Earliest Math HW?
- Did you ever get stuck/unstuck?
- Did a Math Teacher ever make you feel something about math?
- Did a Math Teacher ever make you feel something about yourself?
- Your worst/best math teacher?
- Private math experience?
- Who is the math person in your family?

Questions & Discussion

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$$\frac{2}{3} \cdot \frac{4}{9}$$

$$\frac{3}{10} \cdot \frac{5}{24}$$

$$\frac{2}{7} \cdot \frac{21}{50}$$

$$\frac{3}{8} \cdot 2$$

$$\frac{7}{10} - \frac{4}{10}$$

$$\frac{2}{15} + \frac{8}{15}$$

$$\frac{7}{9} - \frac{4}{9}$$

$$\frac{7}{12} + \frac{7}{12} + \frac{1}{12}$$

Problem Solving Guidelines and Hints

Hint 1: There may be more than one answer.

Hint 2: There may be more than one method/

Hint 3: Use appropriate tools.

Hint 4: Consider simpler, similar problems.

Hint 5: Consider equivalent problems with simpler solutions.

Hint 6: Approximations can be useful.

Hint 7: Try alternative patterns of thought.

Hint 8: Do not spin your wheels.

WELCOME TO FRIDAY!

Prof. Volk

Math 114

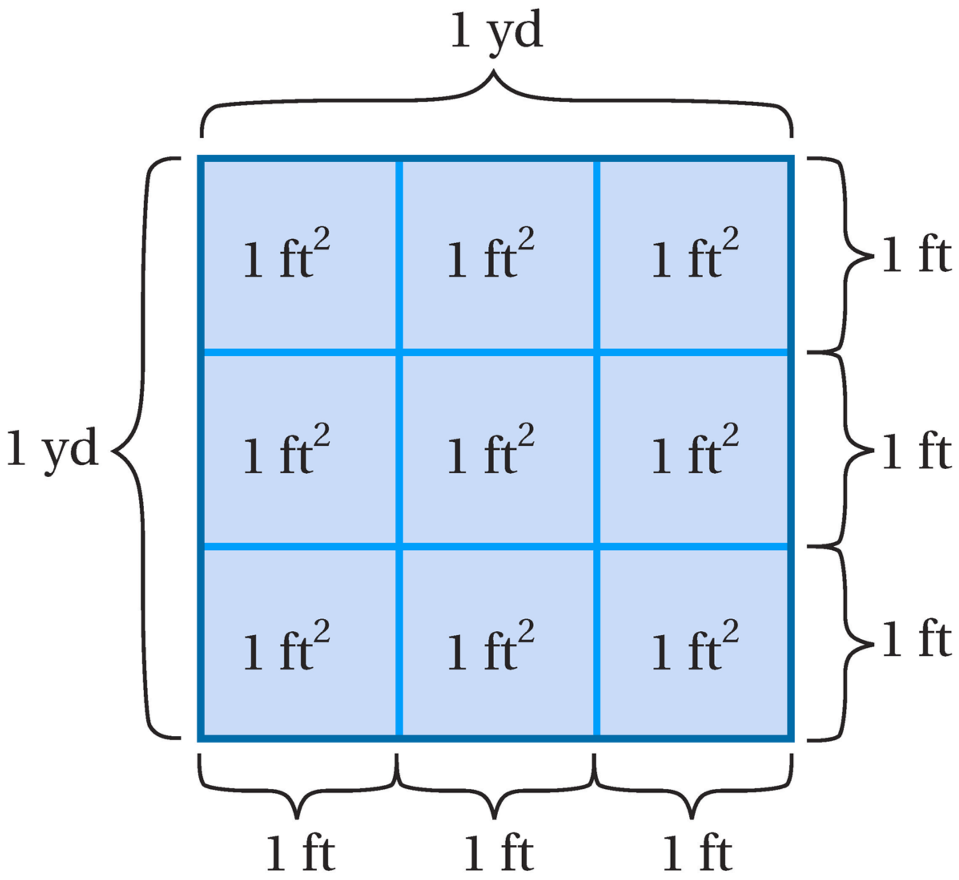
HOW LONG TO READ THE BIBLE?

1. Let's experiment.
2. Read chapter with a timer.
3. Extrapolate!

Conversions with Units Raised to Powers

$$1\text{ yd} = 3\text{ ft}$$

$$\begin{aligned} 1\text{ yd}^2 &= 1\text{ yd} \times 1\text{ yd} \\ &= 3\text{ ft} \times 3\text{ ft} \\ &= 9\text{ ft}^2 \end{aligned}$$



Example: Carpeting a Room

You want to carpet a room that measures 12 feet by 15 feet, making an area of 180 square feet. But carpet is usually sold by the square yard. How many square yards of carpet do you need?

Solution

$$180 \cancel{\text{ ft}^2} \times \frac{1 \text{ yd}^2}{9 \cancel{\text{ ft}^2}} = \frac{180}{9} \text{yd}^2 = 20 \text{ yd}^2$$

Example: Filling a Planter

How many cubic yards of soil are needed to fill a planter that is 20 feet long by 3 feet wide by 4 feet tall?

The volume is $20\text{ ft} \times 3\text{ ft} \times 4\text{ ft} = 240\text{ ft}^3$

$1\text{ yd} = 3\text{ ft}$, so $(1\text{ yd})^3 = (3\text{ ft})^3 = 27\text{ ft}^3$

$$240\text{ ft}^3 \times \frac{1\text{ yd}^3}{27\text{ ft}^3} \approx 8.9\text{ yd}^3$$


LU
ONE


BLAZE PIZZA
FAST. FIRE'D
BLAZE PIZZA
FAST·FIRE'D.
4026-F


ART LOVER
mozzarella
mozzarella
gorgonzola
chicken
red onion
bbq sauce drizzle
1-TOP
FINISHES
pesto
bbq
olive oil
GL FREE
VEG CHZ
TO GO
GC
Sarah


CP CROSSPOINT CHURCH
CP CROSSPOINT CHURCH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HOW MANY MINUTES?

In one week?

Example: Using Metric Prefixes

Convert 2759 centimeters to meters.

$$2759 \text{ cm} \times \frac{1\,\text{m}}{100 \text{ cm}} = 27.59\,\text{m}$$

Example: Using Metric Prefixes

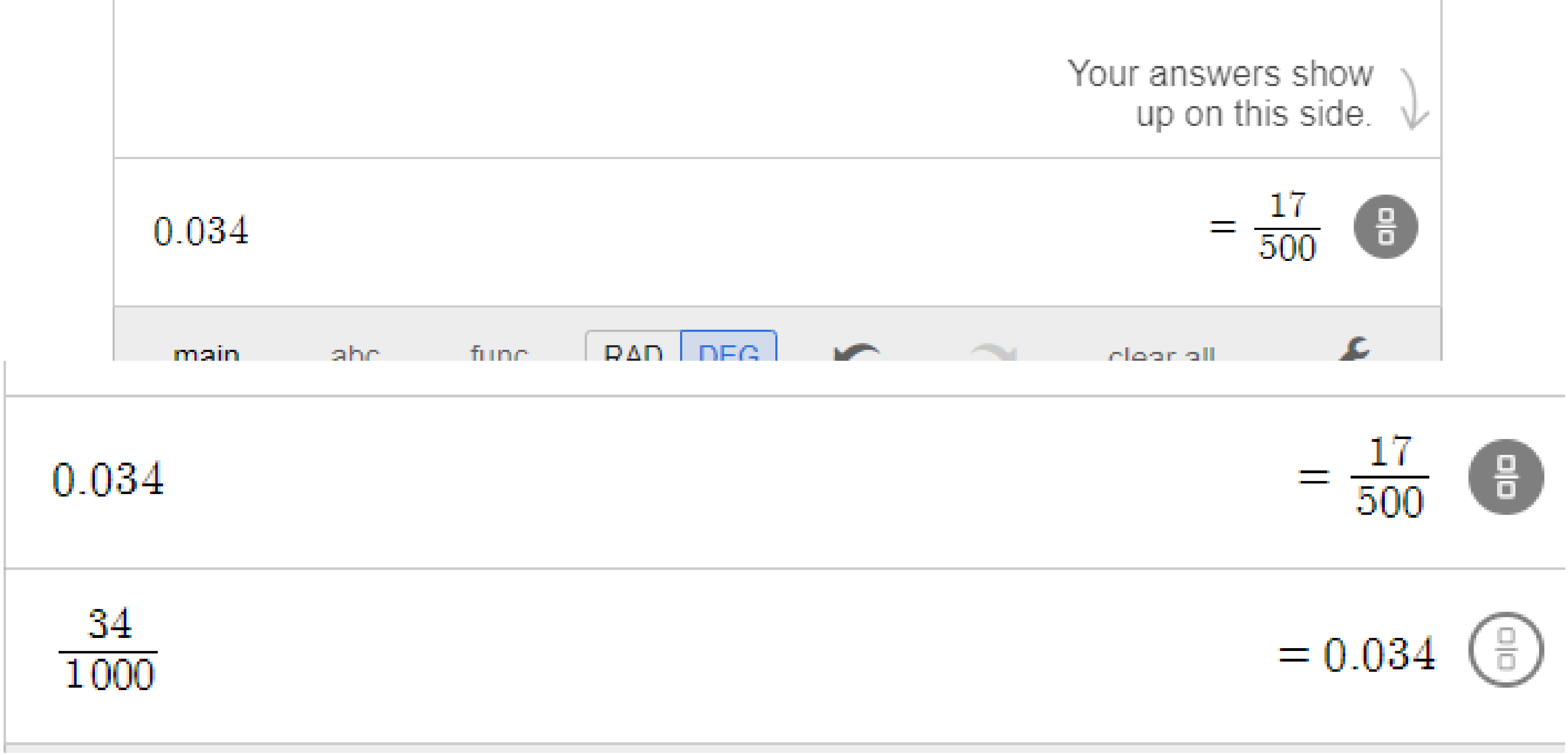
Convert 13562 milligrams to kilograms.

Example: The Kentucky Derby

$$1 \text{ furlong} = \frac{1}{8} \text{ mi}$$

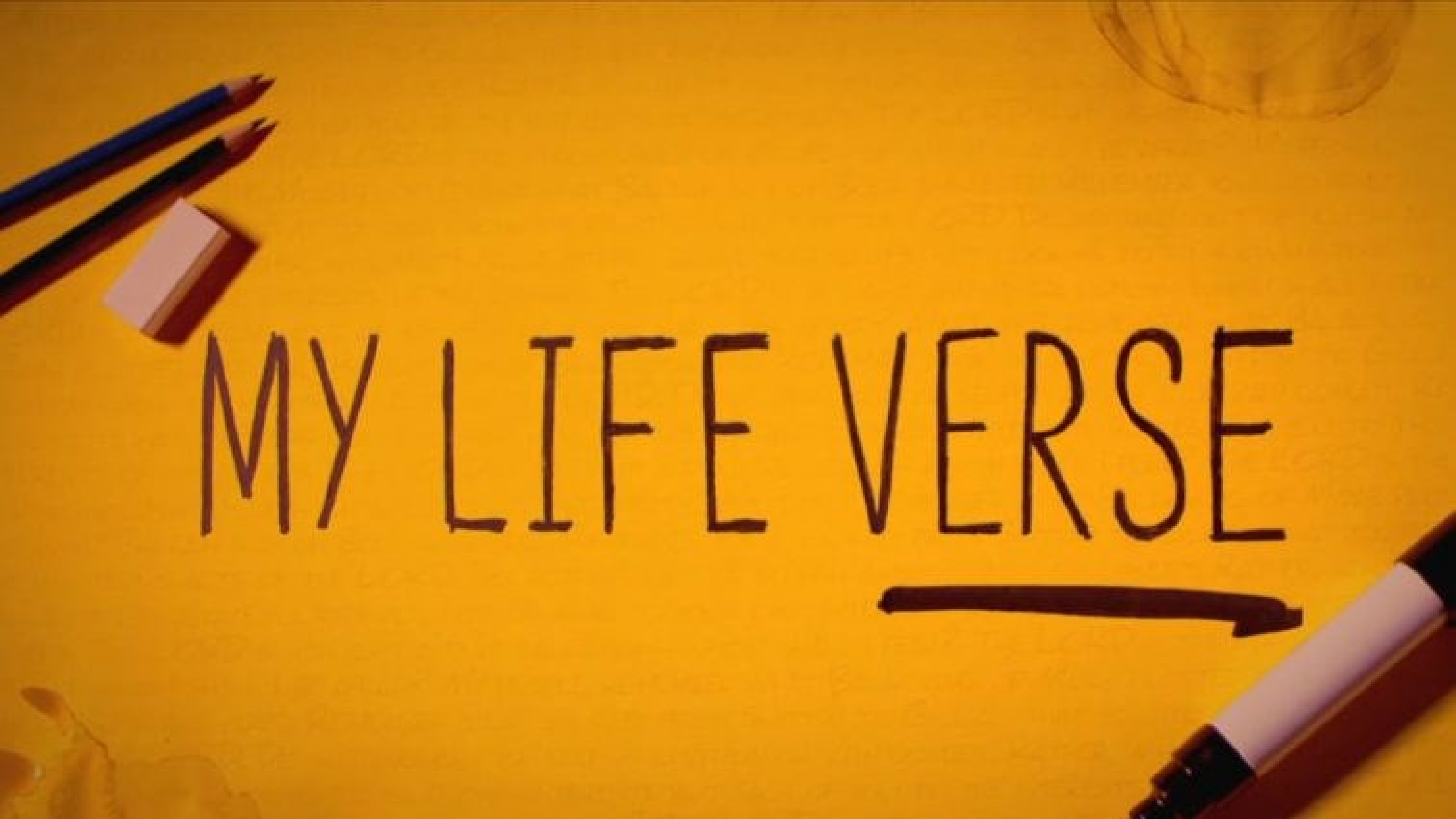
The length of the Kentucky Derby horse race is 10 furlongs.
How long is the race in feet?

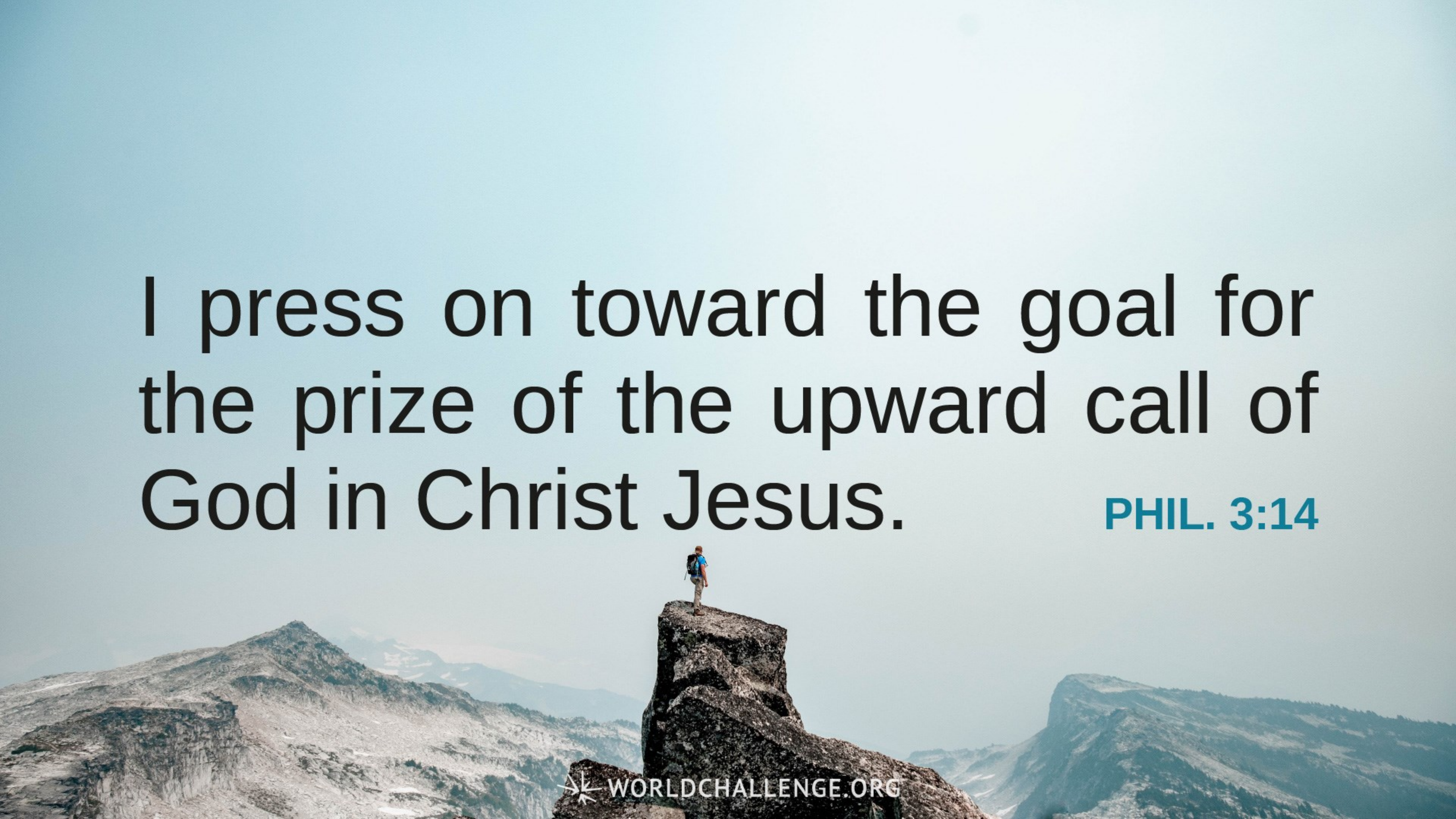
Write 0.034 as a common fraction.



A runner ran 5 kilometers in 18 minutes.
What is their speed in kilometers per hour?

$$\frac{5\ \text{km}}{18\ \text{min}} \times \frac{60\ \text{min}}{1\ \text{hr}} \rightarrow \frac{300\ \text{km}}{18\ \text{hr}} \longrightarrow \frac{16.7\ \text{km}}{\text{hr}}$$


MY LIFE VERSE


I press on toward the goal for the prize of the upward call of God in Christ Jesus.
PHIL. 3:14
WORLDCHALLENGE.ORG

Multiplying Fractions

GUIDED EXAMPLE 1 *Multiply. Write your answer in simplest form.*

$$\frac{2}{3} \cdot \frac{4}{9}$$

SOLUTION

STEP	RESULT
Multiply the numerators.	$2 \cdot 4 = 8$
Multiply the denominators.	$3 \cdot 9 = 27$
Simplify.	$\frac{8}{27}$ does not simplify.
Answer	$\frac{8}{27}$

Multiplying Fractions — Simplifying First

GUIDED EXAMPLE 2 *Multiply by simplifying first.*

$$\frac{3}{10}\cdot\frac{5}{24}$$

SOLUTION

STEP	DETAILS	RESULT
Write the problem as one fraction.	$\frac{3}{10}\cdot\frac{5}{24}=\frac{3\cdot 5}{10\cdot 24}$	$\frac{3\cdot 5}{10\cdot 24}$
Divide 5 and 10 by a common factor of 5.	$\frac{3\cdot \overset{1}{\cancel{5}}}{\underset{2}{\cancel{10}}\cdot 24}$	$\frac{3\cdot 1}{2\cdot 24}$
Divide 3 and 24 by a common factor of 3.	$\frac{\overset{1}{\cancel{3}}\cdot 1}{2\cdot \underset{8}{\cancel{24}}}$	$\frac{1\cdot 1}{2\cdot 8}$
Multiply the remaining factors.	$\frac{1\cdot 1}{2\cdot 8}=\frac{1}{16}$	$\frac{1}{16}$

Multiplying Fractions — Simplifying First

GUIDED EXAMPLE 3 *Multiply by simplifying first.*

$$\frac{2}{7}\cdot\frac{21}{50}$$

SOLUTION

STEP	DETAILS	RESULT
Write the problem as one fraction.	$\frac{2}{7}\cdot\frac{21}{50}=\frac{2\cdot 21}{7\cdot 50}$	$\frac{2\cdot 21}{7\cdot 50}$
Divide 2 and 50 by a common factor of 2.	$\frac{\overset{1}{\cancel{2}}\cdot 21}{7\cdot \underset{25}{\cancel{50}}}$	$\frac{1\cdot 21}{7\cdot 25}$
Divide 7 and 21 by a common factor of 7.	$\frac{1\cdot \overset{3}{\cancel{21}}}{\underset{1}{\cancel{7}}\cdot 25}$	$\frac{1\cdot 3}{1\cdot 25}$
Multiply the remaining factors.	$\frac{1\cdot 3}{1\cdot 25}=\frac{3}{25}$	$\frac{3}{25}$

Multiplying Fractions — Simplifying First

GUIDED EXAMPLE 4 *Multiply by simplifying first.*

$$\frac{3}{8}\cdot 2$$

SOLUTION

STEP	DETAILS	RESULT
Write 2 as a fraction.	$2=\frac{2}{1}$	$\frac{3}{8}\cdot\frac{2}{1}$
Write the problem as one fraction.	$\frac{3}{8}\cdot\frac{2}{1}=\frac{3\cdot 2}{8\cdot 1}$	$\frac{3\cdot 2}{8\cdot 1}$
Divide 2 and 8 by a common factor of 2.	$\frac{3\cdot \overset{1}{\cancel{2}}}{\underset{4}{\cancel{8}}\cdot 1}$	$\frac{3\cdot 1}{4\cdot 1}$
Multiply the remaining factors.	$\frac{3\cdot 1}{4\cdot 1}=\frac{3}{4}$	$\frac{3}{4}$

Adding and Subtracting Like Fractions

GUIDED EXAMPLE 1 *Subtract the following like fractions. Simplify if possible.*

$$\frac{7}{10}-\frac{4}{10}$$

SOLUTION

STEP	RESULT
Are they like fractions?	Yes, they have a common denominator of 10.
Subtract the numerators.	$7-4=3$
Keep the denominator.	$\frac{3}{10}$
Simplify if possible.	$\frac{3}{10}$ is already written in simplest form.
Answer	$\frac{3}{10}$

Adding and Subtracting Like Fractions

GUIDED EXAMPLE 2 *Add the following like fractions. Simplify if possible.*

$$\frac{2}{15}+\frac{8}{15}$$

SOLUTION

STEP	RESULT
Are they like fractions?	Yes, they have a common denominator of 15.
Add the numerators.	$2+8=10$
Keep the denominator.	$\frac{10}{15}$
Simplify if possible.	$\frac{10\div 5}{15\div 5}=\frac{2}{3}$
Answer	$\frac{2}{3}$

Adding and Subtracting Like Fractions

GUIDED EXAMPLE 3 *Subtract the following like fractions. Simplify if possible.*

$$\frac{7}{9}-\frac{4}{9}$$

SOLUTION

STEP	RESULT
Are they like fractions?	Yes, they have a common denominator of 9.
Subtract the numerators.	$7-4=3$
Keep the denominator.	$\frac{3}{9}$
Simplify if possible.	$\frac{3\div 3}{9\div 3}=\frac{1}{3}$
Answer	$\frac{1}{3}$

Adding and Subtracting Like Fractions

GUIDED EXAMPLE 4 *Add the following like fractions. Simplify if possible.*

$$\frac{7}{12}+\frac{7}{12}+\frac{1}{12}$$

SOLUTION

STEP	RESULT
Are they like fractions?	Yes, they have a common denominator of 12.
Add the numerators.	$7+7+1=15$
Keep the denominator.	$\frac{15}{12}$
Simplify if possible.	$\frac{15\div 3}{12\div 3}=\frac{5}{4}$
Answer	$\frac{5}{4}$

Adding Unlike Fractions

GUIDED EXAMPLE 1 *Add. Simplify if possible.* $\frac{3}{7}+\frac{2}{5}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 7 and 5 is 35.
Rewrite $\frac{3}{7}$ with the LCD as the denominator.	$\frac{3\cdot 5}{7\cdot 5}=\frac{15}{35}$
Rewrite $\frac{2}{5}$ with the LCD as the denominator.	$\frac{2\cdot 7}{5\cdot 7}=\frac{14}{35}$
Add the numerators.	$15+14=29$
Keep the denominator.	$\frac{29}{35}$
Simplify if possible.	$\frac{29}{35}$

Adding Unlike Fractions

GUIDED EXAMPLE 2 *Add. Simplify if possible.* $\frac{1}{14}+\frac{3}{7}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 14 and 7 is 14.
Rewrite $\frac{1}{14}$ with the LCD as the denominator.	$\frac{1}{14}$ already has a denominator of 14.
Rewrite $\frac{3}{7}$ with the LCD as the denominator.	$\frac{3\cdot 2}{7\cdot 2}=\frac{6}{14}$
Add the numerators.	$1+6=7$
Keep the denominator.	$\frac{7}{14}$
Simplify if possible.	$\frac{7\div 7}{14\div 7}=\frac{1}{2}$

Adding Unlike Fractions

GUIDED EXAMPLE 3 *Add. Simplify if possible* $\frac{3}{10}+\frac{1}{6}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 10 and 6 is 30.

Notes:

We need to find the LCD of 10 and 6.

Using the List Method:

10: 10,20,30,40,50,…
6: 6,12,18,24,30,36,…

The LCD is 30.

Using the Prime Factor Method:

$$\begin{array}{l} 10 = 2\cdot 5 \\ \underline{\;6 = 2\cdot \quad 3} \\ \quad\;\;\downarrow\;\downarrow\;\downarrow \\ \quad\;\; 2\cdot 5\cdot 3 = 30 = \text{LCD} \end{array}$$

Adding Unlike Fractions

GUIDED EXAMPLE 3 *Add. Simplify if possible* $\frac{3}{10}+\frac{1}{6}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 10 and 6 is 30.
Rewrite $\frac{3}{10}$ with the LCD as the denominator.	$\frac{3\cdot 3}{10\cdot 3}=\frac{9}{30}$

Notes:

To rewrite $\frac{3}{10}$ *with the LCD as the denominator, find the number that you need to multiply the denominator by to get the LCD. We know that the LCD is 30.*

In this case, $10\cdot ?=30$ *means you need to multiply by 3.*

Then multiply the numerator and the denominator by 3.

$$\frac{3\cdot 3}{10\cdot 3}=\frac{9}{30}$$

Adding Unlike Fractions

GUIDED EXAMPLE 3 *Add. Simplify if possible* $\frac{3}{10}+\frac{1}{6}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 10 and 6 is 30.
Rewrite $\frac{3}{10}$ with the LCD as the denominator.	$\frac{3\cdot 3}{10\cdot 3}=\frac{9}{30}$
Rewrite $\frac{1}{6}$ with the LCD as the denominator.	$\frac{1\cdot 5}{6\cdot 5}=\frac{5}{30}$

Notes:

To rewrite $\frac{1}{6}$ with the LCD as the denominator, find the number that you need to multiply the denominator by to get the LCD. We know that the LCD is 30.

In this case, $6\cdot ?=30$ means you need to multiply by 5.

Then multiply the numerator and the denominator by 5.

$$\frac{1\cdot 5}{6\cdot 5}=\frac{5}{30}$$

Adding Unlike Fractions

GUIDED EXAMPLE 3 *Add. Simplify if possible* $\frac{3}{10} + \frac{1}{6}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 10 and 6 is 30.
Rewrite $\frac{3}{10}$ with the LCD as the denominator.	$\frac{3 \cdot 3}{10 \cdot 3} = \frac{9}{30}$
Rewrite $\frac{1}{6}$ with the LCD as the denominator.	$\frac{1 \cdot 5}{6 \cdot 5} = \frac{5}{30}$
Add the numerators.	$9 + 5 = 14$

Notes:

Add the numerators.

$9 + 5 = 14$

Adding Unlike Fractions

GUIDED EXAMPLE 3 *Add. Simplify if possible* $\dfrac{3}{10}+\dfrac{1}{6}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 10 and 6 is 30.
Rewrite $\dfrac{3}{10}$ with the LCD as the denominator.	$\dfrac{3\cdot 3}{10\cdot 3}=\dfrac{9}{30}$
Rewrite $\dfrac{1}{6}$ with the LCD as the denominator.	$\dfrac{1\cdot 5}{6\cdot 5}=\dfrac{5}{30}$
Add the numerators.	$9+5=14$
Keep the denominator.	$\dfrac{14}{30}$

Notes:

Keep the denominator.

$$\frac{9+5}{30}=\frac{14}{30}$$

Adding Unlike Fractions

GUIDED EXAMPLE 3 *Add. Simplify if possible* $\frac{3}{10}+\frac{1}{6}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 10 and 6 is 30.
Rewrite $\frac{3}{10}$ with the LCD as the denominator.	$\frac{3\cdot 3}{10\cdot 3}=\frac{9}{30}$
Rewrite $\frac{1}{6}$ with the LCD as the denominator.	$\frac{1\cdot 5}{6\cdot 5}=\frac{5}{30}$
Add the numerators.	$9+5=14$
Keep the denominator.	$\frac{14}{30}$
Simplify if possible.	$\frac{14\div 2}{30\div 2}=\frac{7}{15}$

Notes:

To simplify, divide the numerator and denominator by the largest number that divides exactly into both. In this case, 14 and 30 can both be divided by 2.

Adding Unlike Fractions

GUIDED EXAMPLE 3 *Add. Simplify if possible* $\frac{3}{10}+\frac{1}{6}$

SOLUTION

STEP	RESULT
Rewrite $\frac{3}{10}$ with the LCD as the denominator.	$\frac{3 \cdot 3}{10 \cdot 3}=\frac{9}{30}$
Rewrite $\frac{1}{6}$ with the LCD as the denominator.	$\frac{1 \cdot 5}{6 \cdot 5}=\frac{5}{30}$
Add the numerators.	$9+5=14$
Keep the denominator.	$\frac{14}{30}$
Simplify if possible.	$\frac{14 \div 2}{30 \div 2}=\frac{7}{15}$
Answer	$\frac{7}{15}$

Notes:

To simplify, divide the numerator and denominator by the largest number that divides exactly into both. In this case, 14 and 30 can both be divided by 2.

Subtracting Unlike Fractions

GUIDED EXAMPLE 4 *Subtract. Simplify if possible.* $\frac{4}{9} - \frac{1}{12}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 9 and 12 is 36.
Rewrite $\frac{4}{9}$ with the LCD as the denominator.	$\frac{4 \cdot 4}{9 \cdot 4} = \frac{16}{36}$
Rewrite $\frac{1}{12}$ with the LCD as the denominator.	$\frac{1 \cdot 3}{12 \cdot 3} = \frac{3}{36}$
Subtract the numerators.	$16 - 3 = 13$
Keep the denominator.	$\frac{13}{36}$
Simplify if possible.	$\frac{13}{36}$ is in simplest form.

Subtracting Unlike Fractions

GUIDED EXAMPLE 5 *Subtract. Simplify if possible.* $\frac{14}{15} - \frac{3}{5}$

SOLUTION

STEP	RESULT
Find the LCD.	The LCD of 15 and 5 is 15.
Rewrite $\frac{14}{15}$ with the LCD as the denominator.	$\frac{14}{15}$ already has a denominator of 15.
Rewrite $\frac{3}{5}$ with the LCD as the denominator.	$\frac{3 \cdot 3}{5 \cdot 3} = \frac{9}{15}$
Subtract the numerators.	$14 - 9 = 5$
Keep the denominator.	$\frac{5}{15}$
Simplify if possible.	$\frac{5 \div 5}{15 \div 5} = \frac{1}{3}$

Application

GUIDED EXAMPLE 6 *Jane ate $\frac{1}{2}$ of a candy bar and Sue ate $\frac{1}{4}$ of the same candy bar. How much more did Jane eat than Sue?*

SOLUTION

Understand the problem.

We want to find how much more Jane ate than Sue.

Create a plan.

We need to find the difference, $\frac{1}{2} - \frac{1}{4}$.

Find the answer.

Find the LCD. The LCD is 4. $\frac{1}{2} - \frac{1}{4}$ becomes $\frac{2}{4} - \frac{1}{4}$.

Subtract. $\frac{2}{4} - \frac{1}{4} = \frac{1}{4}$. Jane ate $\frac{1}{4}$ more of the candy bar than Sue.

Check the answer.

We add the answer plus what Sue ate to equal what Jane ate.

$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$, which simplifies to $\frac{1}{2}$.

Study Guide

!CAUTION Be sure you have a common denominator when adding or subtracting fractions!

***Procedure:* ADDING AND SUBTRACTING UNLIKE FRACTIONS**

1. Find the LCD.
2. Rewrite the fractions as like fractions with the LCD as the denominator.
3. Add or subtract the numerators of the like fractions.
4. Keep the denominator.
5. Simplify if possible.

Application

GUIDED EXAMPLE 5 *There was* $\frac{7}{12}$ *of a lasagna in the refrigerator. Then Geoff ate* $\frac{2}{12}$ *of the lasagna. How much of the lasagna was left?*

SOLUTION

Understand the problem.

We want to find how much of the lasagna remained after Geoff ate.

Create a plan.

We need to subtract the amount Geoff ate from the amount of lasagna in the refrigerator. We need to find $\frac{7}{12} - \frac{2}{12}$.

Find the answer.

$\frac{7}{12} - \frac{2}{12} = \frac{5}{12}$ $\frac{5}{12}$ of the lasagna remained.

Check the answer.

The lasagna was divided into 12 pieces. There were 7 pieces left and Geoff ate 2. This left 5 pieces of lasagna, so $\frac{5}{12}$ of the lasagna remained.

Study Guide

Definition: LIKE FRACTIONS

Fractions with the same, or common, denominator are called like fractions.

Definition: UNLIKE FRACTIONS

Fractions without a common denominator are called unlike fractions.

Procedure: ADDING AND SUBTRACTING LIKE FRACTIONS

1. Add or subtract the numerators.
2. Keep the denominator.
3. Simplify if possible.

Application

GUIDED EXAMPLE *A recent survey shows that* $\frac{3}{4}$ *of graduating seniors plan to go to college. Of those planning to attend, only* $\frac{2}{5}$ *will actually enroll. What fraction of graduating seniors will actually enroll in college?*

SOLUTION

Understand the problem.

We need to find the fraction of seniors that will enroll in college.

Create a plan.

$\frac{3}{4}$ plan to go but only $\frac{2}{5}$ of them will enroll. We need to find of $\frac{2}{5}$ or $\frac{3}{4}$. $\frac{2}{5}\cdot\frac{3}{4}$

Check the answer. Is this what the problem asks for?

Find the answer.

Yes, the problem asks for the fraction that will enroll in college.

$\frac{2}{5}\cdot\frac{3}{4}=\frac{2\cdot 3}{5\cdot 4}$ Divide 2 and 4 by a common factor of 2. $\frac{\overset{1}{\cancel{2}}\cdot 3}{5\cdot \underset{2}{\cancel{4}}}=\frac{1\cdot 3}{5\cdot 2}=\frac{3}{10}$

So, $\frac{3}{10}$ of graduating seniors will enroll in college.

Study Guide

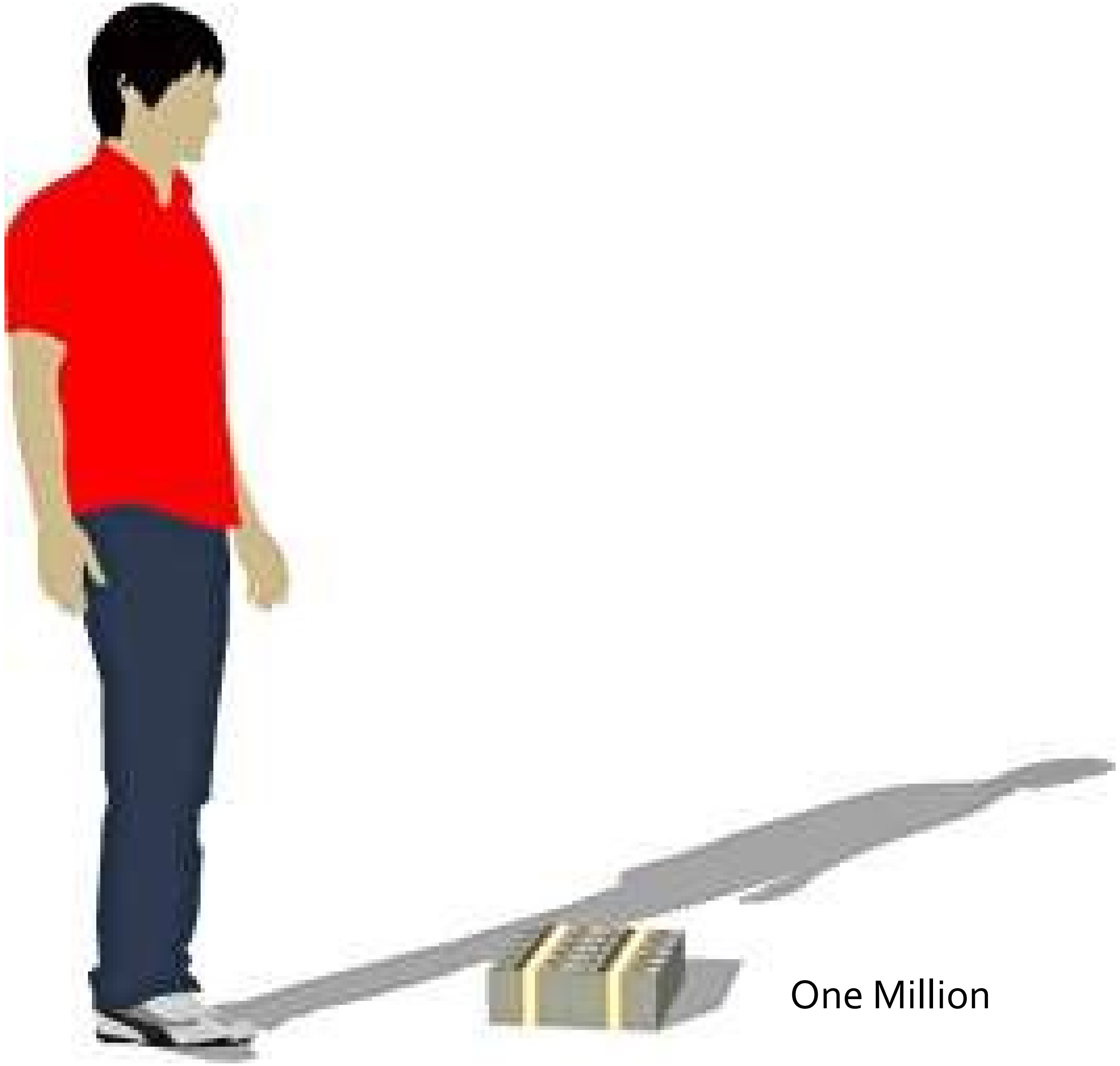
Procedure: MULTIPLYING FRACTIONS

1. Multiply the numerators to get the numerator of the product.
2. Multiply the denominators to get the denominator of the product.
3. Simplify.

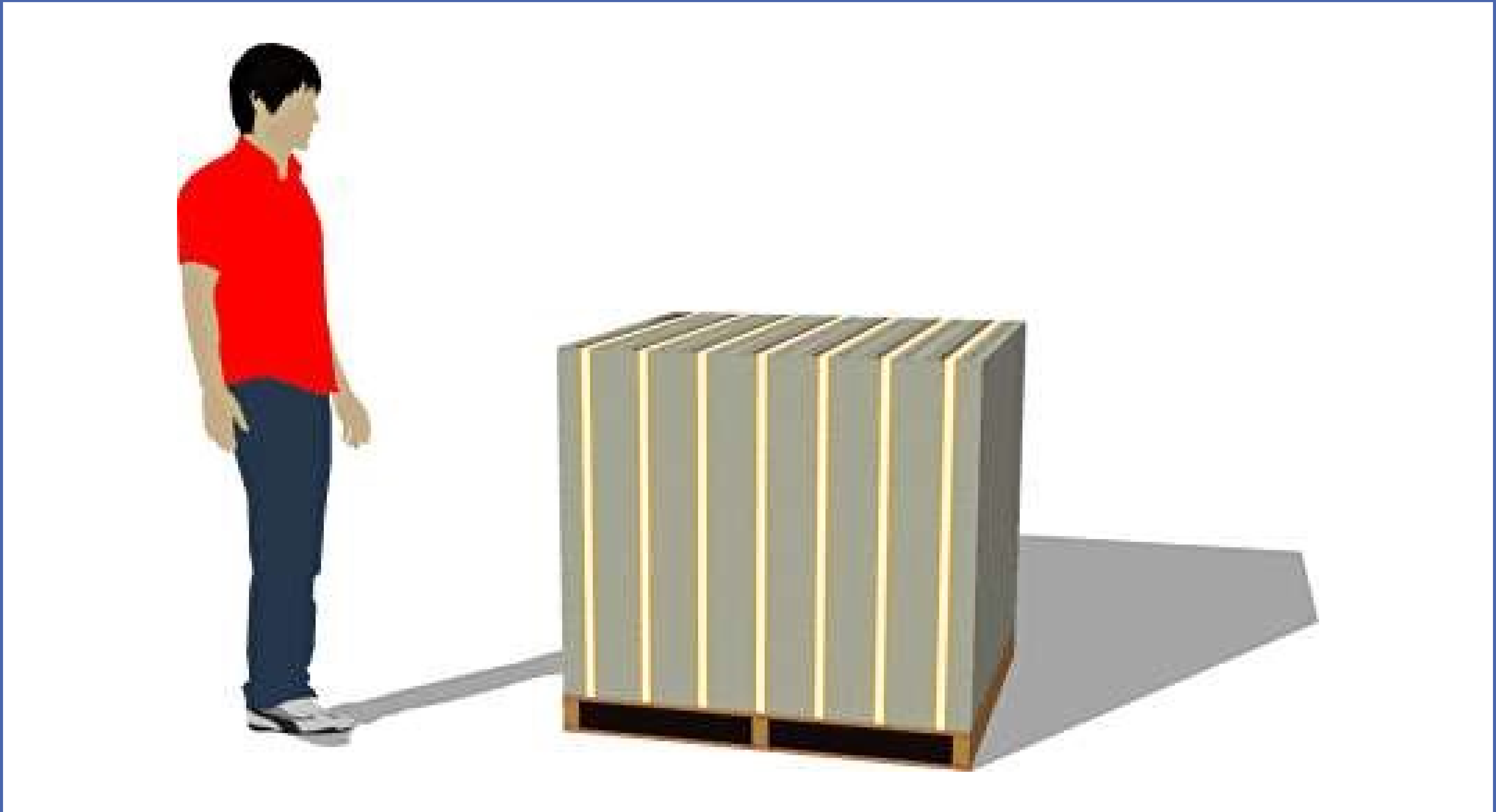
Procedure: MULTIPLYING FRACTIONS — SIMPLIFYING FIRST

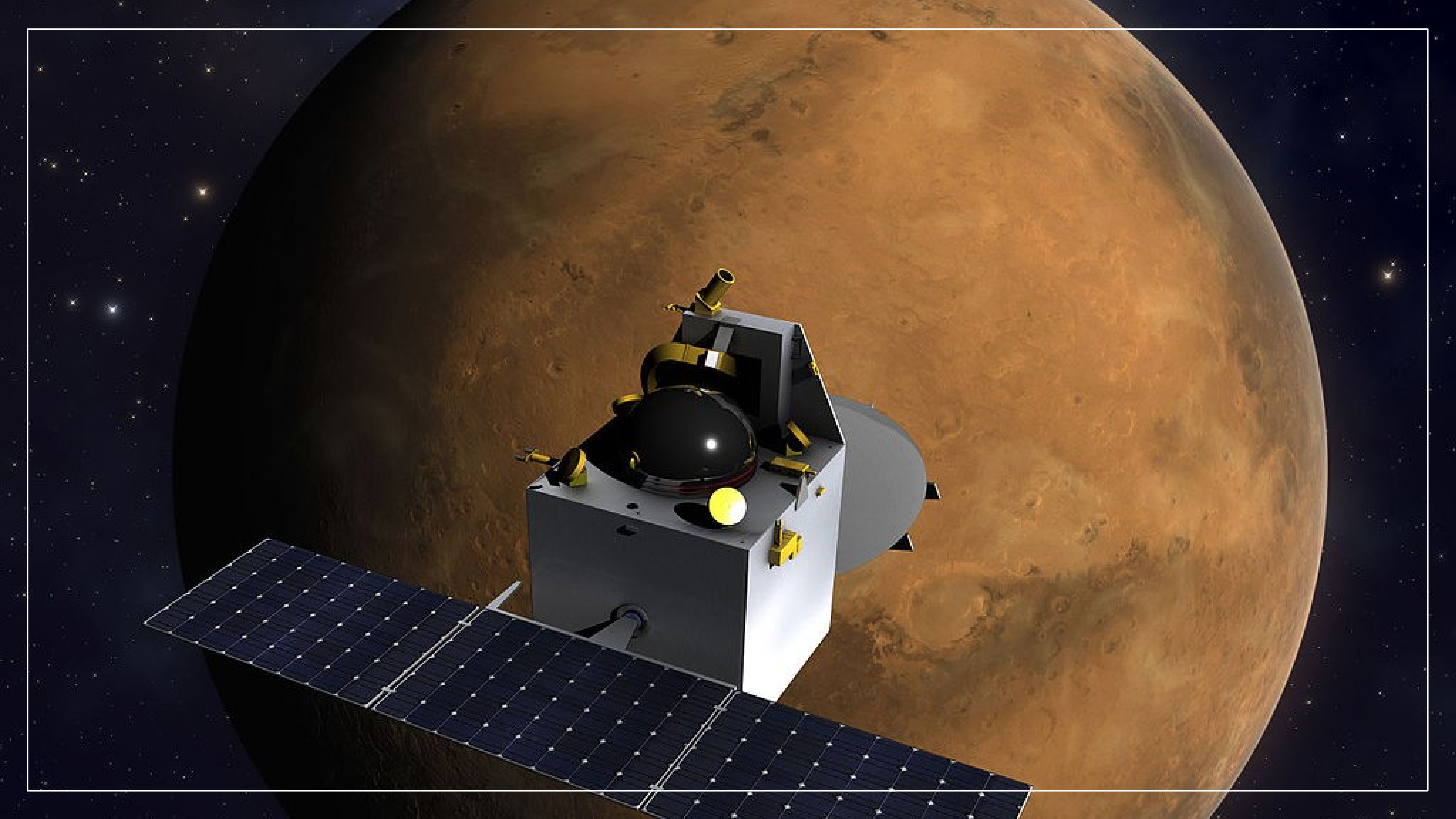
1. Write the problem as one fraction. Do not multiply yet.
2. Divide by common factors in the numerator and denominator.
3. Multiply the remaining factors in the numerator and denominator.


AE 77665544 B
E5
100
$10,000
$10,000
AE 77665544 B
100
HUNDRED DOLLARS
$10,000
$10,000



One Million





Unit 2A Understand, Solve and Explain

Example 1: Using Key Words (1 of 2)

You are buying 30 acres of farm land at $12,000 per acre. What is the total cost?

Solution

Understand. The question asks about total cost, and one of the given units is dollars, so we expect an answer in dollars.

Example 1: Using Key Words (2 of 2)

Solve: We carry out the calculation; note that the price is given in dollars per acre, so we write the division by acres in fraction form. That allows “acres” to cancel, leaving the final answer in dollars:

$$30 \text{ acres } \times \frac{\$12,000}{\text{acre}} = \$360,000$$

Explain: We have found that purchasing 30 acres of farmland at a price of $12,000 per acre will cost a total of $360,000.

Unit Analysis in Problem Solving

Remember:

- You cannot add or subtract numbers with different units, but you can combine different units through multiplication, division, or raising to powers.
- It is easier to keep track of units if you replace division with multiplication by the reciprocal. For example, instead of dividing by 60 s/min, multiply by 1 min/60 s.
- When you complete your calculations, make sure that your answer has the units you expected. If it doesn’t, then you’ve done something wrong.

Key Words and Operations with Units

Key word or symbol	Operation	Example
per	Division	Read miles ÷ hours as "miles per hour."
of or hyphen	Multiplication	Read kilowatts × hours as "kilowatt-hours."
square	Raising to second power	Read $\text{ft} \times \text{ft}$, or ft^2, as "square feet" or "feet squared."
cube or cubic	Raising to third power	Read $\text{ft} \times \text{ft} \times \text{ft}$, or ft^3, as "cubic feet" or "feet cubed."

Example: Using Units to Find a Pathway

How many crates do you need to hold 2000 apples if each crate holds 40 apples?

Solution

The question asks "how many crates," so the answer should have units of *crates*. Notice that the statement "each crate holds 40 apples" implies that we can fit 40 apples *per* crate, which we write as
40 apples/crate.

$$2000\ \text{apples} \div \frac{40\ \text{apples}}{\text{crate}}$$

$$= 2000\ \cancel{\text{apples}} \div \frac{1\ \text{crate}}{40\ \cancel{\text{apples}}} = 50\ \text{crates}$$

Conversion Factors

A **conversion factor** is a statement of equality that is used to convert between units.

Some conversion factors:

$$12 \text{ in.} = 1 \text{ ft} \quad \text{or} \quad \frac{12 \text{ in.}}{1 \text{ ft}} = 1 \quad \text{or} \quad \frac{1 \text{ ft}}{12 \text{ in.}} = 1$$

$$24 \text{ hr} = 1 \text{ day} \quad \text{or} \quad \frac{24 \text{ hr}}{1 \text{ day}} = 1 \quad \text{or} \quad \frac{1 \text{ day}}{24 \text{ hr}} = 1$$

Example: Feet to Inches

Convert a distance of 9 feet into inches.

$$9\ \text{ft} = 9\ \cancel{\text{ft}} \times \frac{12\ \text{in.}}{1\ \cancel{\text{ft}}} = 108\ \text{in.}$$

Example: Using a Chain of Conversions

How many seconds are in 94 days?

Solution

1 day = 24 hours

1 hour = 60 minutes

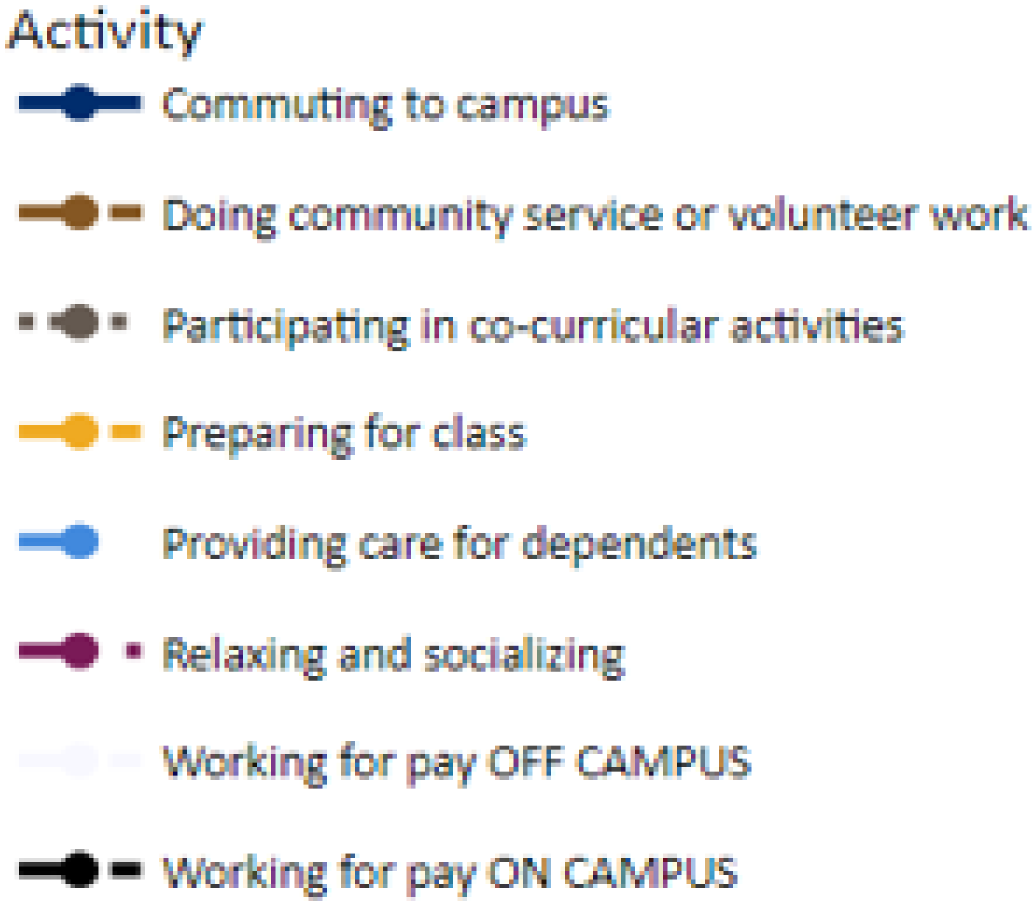
1 minute = 60 seconds

$$94\ \cancel{\text{days}} \times \frac{24\ \cancel{\text{hr}}}{1\ \cancel{\text{day}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{hr}}} \times \frac{60\ \text{sec}}{1\ \cancel{\text{min}}} = 8{,}121{,}600\ \text{sec}$$

How are College Students Spending their Time?

Assign a number between 0-168 to each of the tasks.

How many hours per week do you spend doing each task on average?



Currency Conversions

Converting between currencies is a unit conversion problem in which the conversion factors are known as the exchange rates. The table represents some typical currency exchange rates:

Sample Currency Exchange Rates

Currency	Dollars per Foreign	Foreign per Dollar
British pound	1.624	0.6158
Canadian dollar	1.005	0.9950
European euro	1.320	0.7576
Japanese yen	0.0120	83.33
Mexican peso	0.07855	12.73

Example: Gas Price per Liter (1 of 2)

A gas station in Canada sells gasoline for CAD 1.34 per liter. (CAD is an abbreviation for Canadian dollars.) What is the price in dollars per gallon? Use the currency exchange rate in Table 2.4.

Solution

We use a chain of conversions to convert from CAD to dollars and then from liters to gallons. From Table 2.4, the currency conversion is $1.005 per CAD, and from Table 2.3, there are 3.785 liters per gallon.

Example: Gas Price per Liter

A gas station in Canada sells gasoline for CAD 1.34 per liter. (CAD is an abbreviation for Canadian dollars.) What is the price in dollars per gallon? Use the currency exchange rate in Table 2.4.

$$\frac{1.34\ \cancel{\text{CAD}}}{1\ \cancel{\text{L}}} \times \frac{\$1.005}{1\ \cancel{\text{CAD}}} \times \frac{3.785\ \cancel{\text{L}}}{1\ \text{gal}} \approx \frac{\$5.10}{1\ \text{gal}}$$

Use the sample currency exchange rates given in the table to answer the following question. State all of the conversion factors that you use.

Suppose that a new fuel-efficient European car travels an average of 23 kilometers on 1 liter of gasoline. If gasoline costs 1.25 euros per liter, how much will it cost to drive 200 kilometers in dollars?

Currency	Dollars per Foreign	Foreign per Dollar
British pound	1.414	0.7072
Canadian dollar	0.7834	1.277
European euro	1.256	0.7965
Japanese yen	0.01007	99.34
Mexican peso	0.06584	15.19

$$\frac{\text{1 Liter}}{\text{23 km}} \times \frac{\text{1.25 euro}}{\text{1 Liter}} \times \frac{\text{1.256 USD}}{\text{1 euro}} \rightarrow \frac{\text{1.57 USD}}{\text{23 km}} \times \frac{8.696}{8.696} \rightarrow \frac{\boxed{\text{14.08 Dollars}}}{\text{200 km}}$$

200/23= 8.696

Unit 2B Extending Unit Analysis

U.S. Customary System

The U.S. customary system has roots dating back thousands of years, and its units became standardized in often surprising ways. For example, the modern length of 1 yard was defined by English King Henry I (1100–1135), who decreed it to be the distance from the tip of *his* nose to the tip of *his* thumb on *his* outstretched arm.

The table in your text summarizes the official U.S. customary system, showing standard units for length, weight, and volume.

Example: The Kentucky Derby (1 of 2)

The length of the Kentucky Derby horse race is 10 furlongs. How long is the race in miles?

Solution

See page 78 for Table 2.1, $1 \text{ furlong} = \frac{1}{8} \text{ mi}$

which is the same as 0.125 mile. We can write the conversion factor in two other equivalent forms:

$$\frac{1 \text{ furlong}}{0.125 \text{ m}} = 1 \text{ or } \frac{0.125 \text{ mi}}{1 \text{ furlong}} = 1$$

Example: The Kentucky Derby (2 of 2)

The length of the Kentucky Derby horse race is 10 furlongs. How long is the race in miles?

$$10 \ \cancel{\text{furlong}} \times \frac{0.125 \ \text{mi}}{1 \ \cancel{\text{furlong}}} = 1.25 \ \text{mi}$$

The Kentucky Derby is a race of 1.25 miles.

Example: Price Comparison (1 of 2)

You are planning to make pesto and need to buy basil. At the grocery store, you can buy small containers of basil priced at \$2.99 for each $\frac{2}{3}$-ounce container.

At the farmer's market, you can buy basil in bunches for \$12 per pound. Which is the better deal?

Solution

To compare the prices, we need them both in the same units.

Convert the small container price to a price per pound.

Example: Price Comparison (2 of 2)

The container price is $\$2.99 \; per \; \frac{2}{3}$ ounce, which means we need to divide. We then multiply by the conversion of 16 ounces per pound:

$$\frac{\$2.99}{\frac{2}{3}\ \text{oz}} \times \frac{16\ \text{oz}}{1\ \text{lb}} = \frac{\$71.76}{\text{lb}}$$

The small containers are priced at almost $72 per pound, which is six times as much as the farmer’s market price.

Metric System

The international metric system was invented in France late in the 18th century for two primary reasons: (1) to replace many customary units with just a few basic units and (2) to simplify conversions through use of a decimal (base 10) system. The basic units of length, mass, time, and volume in the metric system are

- the **meter** for length, abbreviated m
- the **kilogram** for mass, abbreviated kg
- the **second** for time, abbreviated s
- the **liter** for volume, abbreviated L

Common Metric Prefixes

Common Metric Prefixes

Small Values

Prefix	Abbrev.	Value
deci	d	10^{-1} (one-tenth)
centi	c	10^{-2} (one-hundredth)
milli	m	10^{-3} (one-thousandth)
micro	μ or mc*	10^{-6} (one-millionth)
nano	n	10^{-9} (one-billionth)
pico	p	10^{-12} (one-trillionth)

Large Values

Prefix	Abbrev.	Value
deca	da	10^{1} (ten)
hecto	h	10^{2} (hundred)
kilo	k	10^{3} (thousand)
mega	M	10^{6} (million)
giga	G	10^{9} (billion)
tera	T	10^{12} (trillion)

* Micro is usually abbreviated with μ (Greek letter mu), but in medical fields it is common to use “mc” intead.

Example: Using Metric Prefixes

Convert 2759 centimeters to meters.

Solution

The table shows that centi means 10^{-2} so $1 \text{ cm} = 10^{-2} \text{ m}$ or, equivalently, 1 m = 100 cm. Therefore, 2759 centimeters is the same as

$$2759 \text{ cm} \times \frac{1 \text{m}}{100 \text{ cm}} = 27.59 \text{ m}$$

Example: Marathon Distance

The marathon running race is about 26.2 miles. About how far is it in kilometers?

Solution

Table 2.3 shows that 1 mi = 1.6093 km. We use the conversion in the form with miles in the denominator to find

$$26.2\ \text{mi} \times \frac{1.6093\ \text{km}}{1\ \text{mi}} = 42.2\ \text{km}$$

Temperature Units (1 of 3)

The **Fahrenheit** scale, commonly used in the United States, is defined so water freezes at 32°F and boils at 212°F.

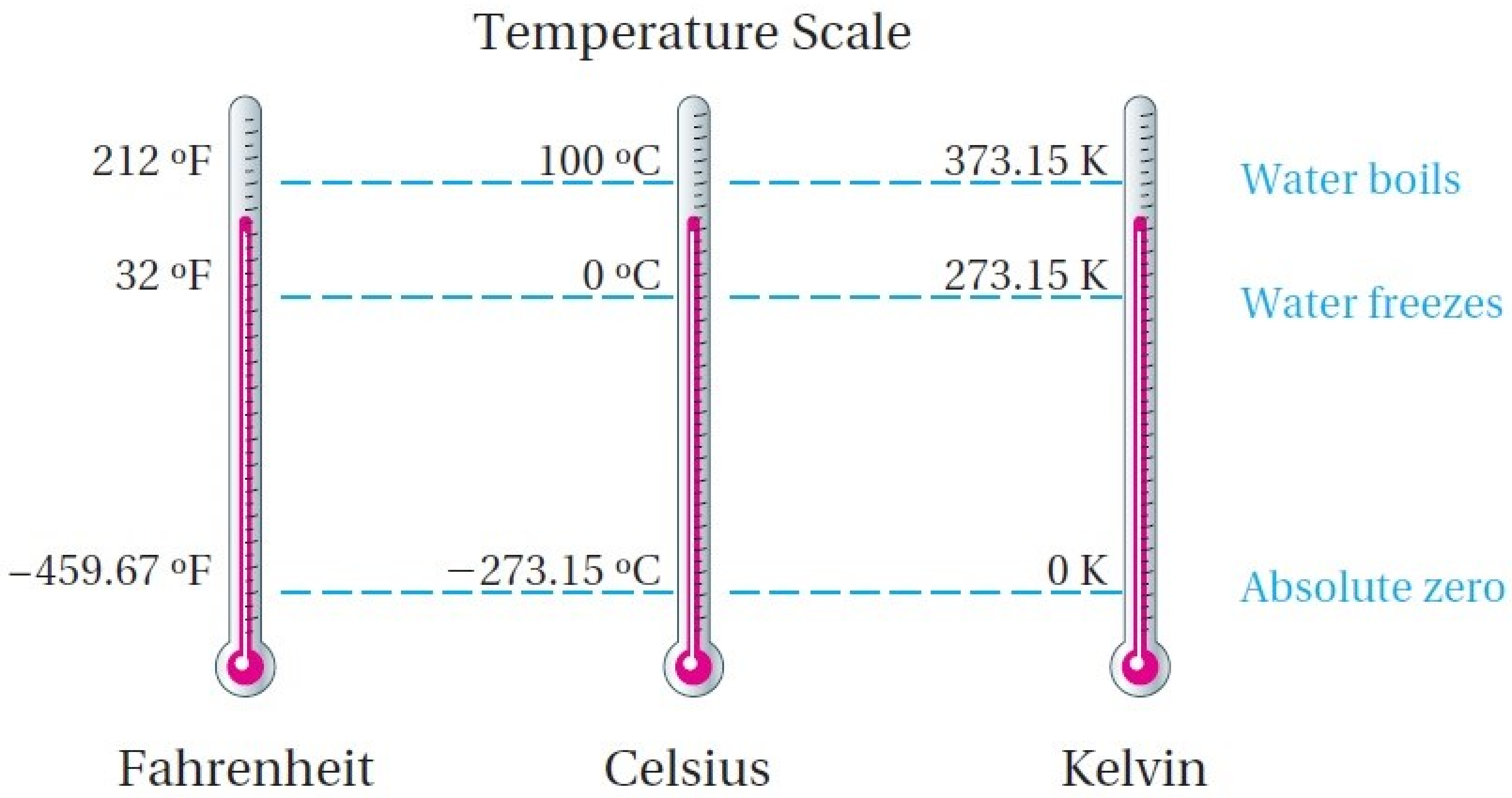
The rest of the world uses the **Celsius** scale, which places the freezing point of water at 0°C and the boiling point at 100°C.

Temperature Units (2 of 3)

In science, we use the **Kelvin** scale, which is the same as the Celsius scale except for its zero point, which corresponds to −°273.15°C. A temperature of 0 K is known as **absolute zero**, because it is the coldest possible temperature. (The degree symbol

$[\circ]$ is not used on the Kelvin scale.)

Temperature Units (3 of 3)



Temperature Conversions

The conversions are given both in words and with formulas in which *C*, *F*, and *K* are Celsius, Fahrenheit, and Kelvin temperatures, respectively.

To Convert from	Conversion in Words	Conversion Formula
Celsius to Fahrenheit	Multiply by 1.8 (or $\frac{9}{5}$). Then add 32.	$F = 1.8C + 32$
Fahrenheit to Celsius	Subtract 32. Then divide by 1.8, which is $\frac{9}{5}$, or equivalently multiply by $\frac{5}{9}$.	$C = \frac{F - 32}{1.8}$
Celsius to Kelvin	Add 273.15.	$K = C + 273.15$
Kelvin to Celsius	Subtract 273.15.	$C = K - 273.15$

Example: Human Body Temperature

Average human body temperature is 98.6°F. What is it in Celsius and Kelvin?

Solution

Convert from Fahrenheit to Celsius by subtracting 32 and then dividing by 1.8:

$$C = \frac{F - 32}{1.8} = \frac{98.6 - 32}{1.8} = \frac{66.6}{1.8} = 37.0^{\circ}\mathrm{C}$$

We find the Kelvin equivalent by adding 273.15 to the Celsius temperature:

$$K = C + 273.15 = 37 + 273.15 = 310.15\ \mathrm{K}$$

Units of Energy and Power

Energy is what makes matter move or heat up. International metric unit is the **joule.**

Power is the rate at which energy is used. International metric unit is the **watt.**

$$1 \text{ watt} = 1 \frac{\text{joule}}{\text{s}}$$

A **kilowatt-hour** is a unit of energy.

1 kilowatt-hour = 3.6 million joules

Example: Operating Cost of a Light Bulb (1 of 2)

A utility company charges $15¢$ per kilowatt-hour of electricity. How much does it cost to keep a 100-watt light bulb on for a week? How much will you save in a year if you replace the bulb with an LED bulb that provides the same amount of light for only 25 watts of power?

Solution

$$100\ \cancel{\text{watt}} \times \frac{1\ \text{kilowatt}}{1000\ \cancel{\text{watt}}} \times 1\ \cancel{\text{week}} \times \frac{7\ \cancel{\text{day}}}{1\ \cancel{\text{week}}} \times \frac{24\ \text{hr}}{1\ \cancel{\text{day}}}$$

$$= 16.8\ \text{Kilowatt - hour}$$

Example: Operating Cost of a Light Bulb (2 of 2)

Now find the cost.

$$16.8\,\cancel{\text{kilo}}\text{watt-hour} \times 15\,\frac{\text{cents}}{\cancel{\text{kilo}}\text{watt-hr}} = 252 = \$2.52$$

The electricity for the bulb costs \$2.52 per week. If you replace the 100-watt bulb with a 25-watt LED, you'll use only

$\frac{1}{4}$ as much energy, which means your weekly cost will be

only $63¢.$ In other words, your savings will be \$2.52 − \$0.63

= \$1.89 per week, so in a year you'll save about:

$$\frac{\$1.89}{\text{wk}} \times \frac{52\text{ wk}}{\text{yr}} \approx \$98/\text{yr}$$

Unit 2C Problem-Solving Hints

Problem Solving Guidelines and Hints

Hint 1: There may be more than one answer.

Hint 2: There may be more than one method/

Hint 3: Use appropriate tools.

Hint 4: Consider simpler, similar problems.

Hint 5: Consider equivalent problems with simpler solutions.

Hint 6: Approximations can be useful.

Hint 7: Try alternative patterns of thought.

Hint 8: Do not spin your wheels.

Example: Box Office Receipts (1 of 3)

Tickets for a fundraising event were priced at \$10 for children and \$20 for adults. Shauna worked the first shift at the box office, selling a total of \$130 worth of tickets. However, she did not keep a careful count of how many tickets she sold for children and adults. How many tickets of each type (child and adult) did she sell?

Solution

Try trial and error. Suppose Shauna sold just one \$10 child ticket. In that case, she would have sold \$130 − \$10 = \$120 worth of adult tickets.

Example: Box Office Receipts (2 of 3)

Because the adult tickets cost \$20 apiece, this means she would have sold $\$120 \div (\$20 \text{ per adult ticket}) = 6 \text{ adult tickets.}$

We have found an answer to the question: Shauna could have collected \$130 by selling 1 child and 6 adult tickets. But it is not the only answer, as we can see by testing other values.

For example, suppose she sold three of the \$10 child tickets, for a total of \$30. Then she would have sold \$130 − \$30 = \$100 worth of adult tickets, which means 5 of the \$20 adult tickets.

Example: Box Office Receipts (3 of 3)

We have a second possible answer—3 child and 5 adult tickets—and have no way to know which answer is the actual number of tickets sold. In fact, there are seven possible answers to the question. In addition to the two answers we’ve already found, other possible answers are 5 child tickets and 4 adult tickets; 7 child tickets and 3 adult tickets; 9 child tickets and 2 adult tickets; 11 child tickets and 1 adult ticket; and 13 child tickets with 0 adult tickets. Without further information, we do not know which combination represents the actual ticket sales.

Example: Jill and Jack’s Race (1 of 4)

Jill and Jack ran a 100-meter race. Jill won by 5 meters; that is, Jack had run only 95 meters when Jill crossed the finish line. They decide to race again, but this time Jill starts 5 meters behind the starting line. Assuming that both runners run at the same pace as before, who will win?

(see solution next slides)

Example: Jill and Jack's Race (2 of 4)

Solution Method 1: One approach to this problem is analytical—we analyze each race quantitatively. We were not told how fast either Jill or Jack ran, so we can choose some reasonable numbers. For example, we might assume that Jill ran the 100 meters in the first race in 20 seconds. In that case, her pace was 100 m ÷ 20 s = 5 meters per second (5 m/s). Because Jack ran only 95 meters in the same 20 seconds, his pace was 95 m ÷ 20 s = 4.75 m/s.

Example: Jill and Jack's Race (3 of 4)

For the second race, Jill must run 105 meters (because she starts 5 meters behind the starting line) to Jack's 100 meters. We predict their times by dividing their respective race distances by their speeds from the first race:

Jill $$105\text{ m} \div 5\frac{\text{m}}{\text{s}} = 105\text{ m} \times \frac{1\text{ s}}{5\text{ m}} = 21\text{ s}$$

Jack $$100\text{ m} \div 4.75\frac{\text{m}}{\text{s}} = 100\text{ m} \times \frac{1\text{ s}}{4.75\text{ m}} \approx 21.05\text{ s}$$

Example: Jill and Jack’s Race (4 of 4)

Solution Method 2: Although the analytical method works, we can use a much more intuitive and direct solution. In the first race, Jill runs 100 meters in the same time that Jack runs 95 meters. Therefore, in the second race, Jill will pull even with Jack 95 meters from the starting line. In the remaining 5 meters, Jill’s faster speed will allow her to pull away and win. Note how this insight avoids the calculations needed for the analytical method.

QUESTIONS?