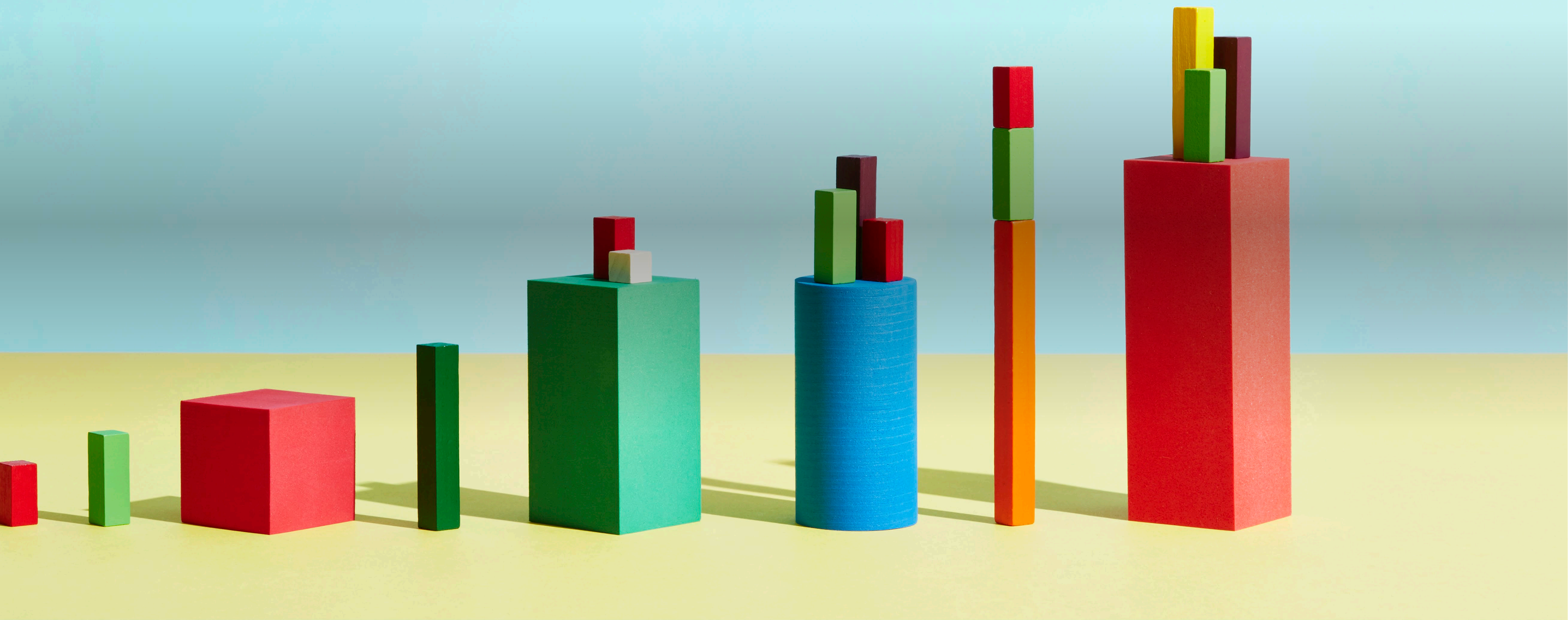
Math 114 Module 11: Growth

Prof. Volk

Spring 2026



Homework 11: 20 Questions

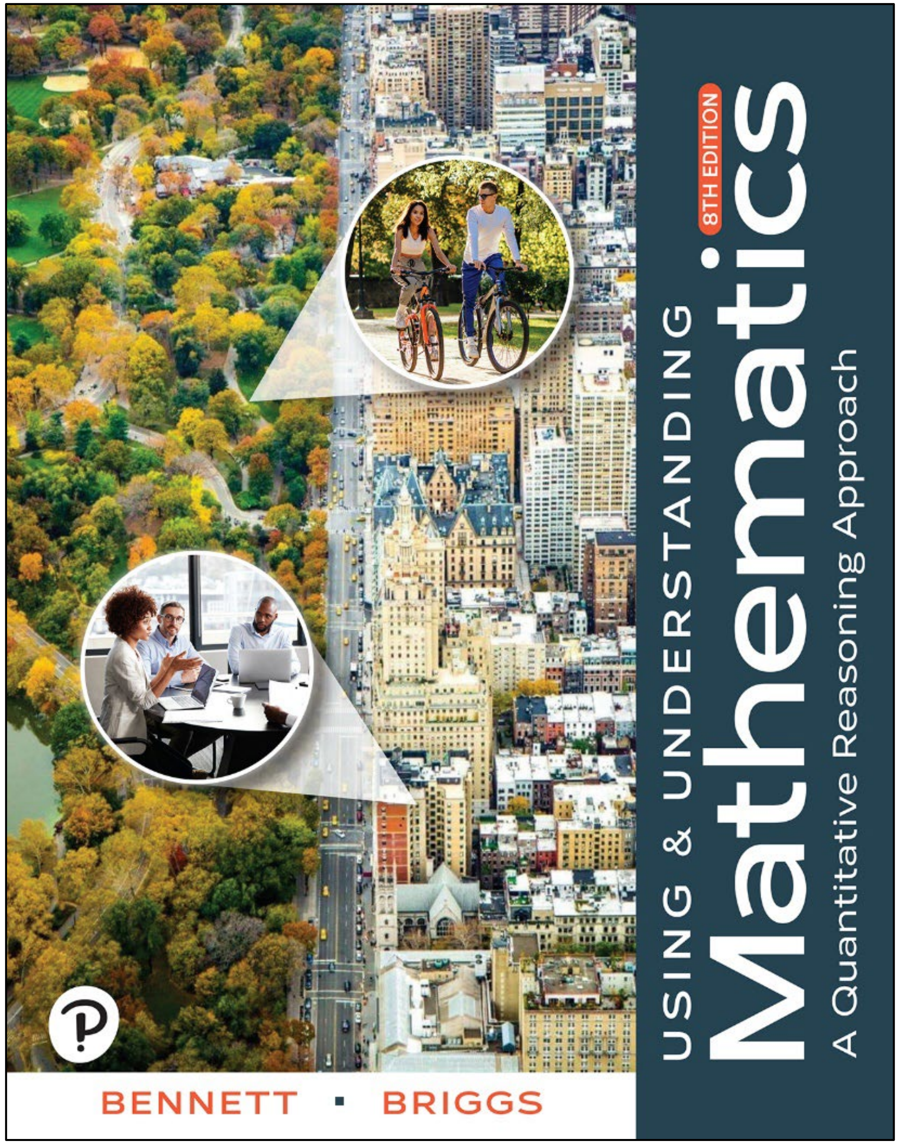
Due Sunday 11:59 pm

Questions 1 - 4	• Distinguish between linear and exponential changes and find results of given changes.
Question 5	• Solve application problems involving linear and exponential growth and decay.
Questions 6 - 7	• Give examples and explain concepts and terms related to linear and exponential change.
Questions 8 - 9	• Decide whether statements related to linear and exponential change make sense.
Questions 10 - 12	• Find growth factors, times, doubling times, and future quantities for exponential growth.
Questions 13 - 15	• Decide if the approximate doubling time formula is valid for a given case, and use it if it is.
Questions 16 - 18	• Find decay factors, times, half-lives, and future quantities for exponential decay.
Questions 19	• Approximate and evaluate logarithms.
Question 20	• Perform rate calculations related to real population growth.

Quiz 11: 10 Questions

Due Monday 11:59 pm

Question 1	• Distinguish between linear and exponential changes and find results of given changes.
Question 2	• Solve application problems involving linear and exponential growth and decay.
Question 3	• Give examples and explain concepts and terms related to linear and exponential change.
Question 4	• Decide whether statements related to linear and exponential change make sense.
Questions 5 - 6	• Find growth factors, times, doubling times, and future quantities for exponential growth.
Question 7	• Decide if the approximate doubling time formula is valid for a given case, and use it if it is.
Question 8	• Find decay factors, times, half-lives, and future quantities for exponential decay.
Question 9	• Approximate and evaluate logarithms.
Question 10	• Perform rate calculations related to real population growth.



This Week's Reading

Chapter 8

(Sections A, B, & C)

Linear and Exponential Growth

Growth Models

Linear, Exponential, and Logistic

Module 11

Linear

Adds the same amount each time. Constant difference.

Exponential

Multiplies by the same factor each time. Constant percent change.

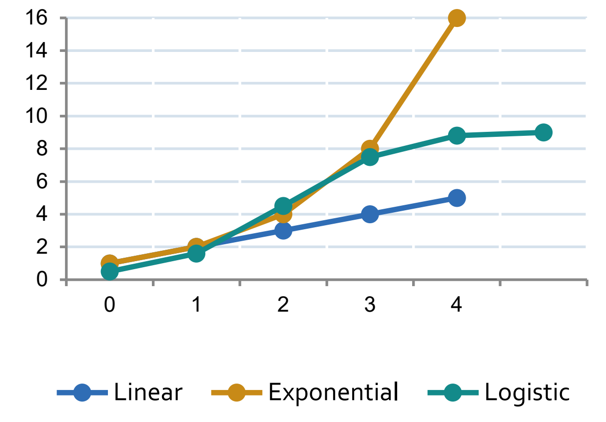
Logistic

Begins quickly but slows as limits push the population toward carrying capacity.

What students should know

- How to identify each model from words, tables, and graphs
- How to compute exponential growth and doubling time
- Why carrying capacity changes a curve from J-shaped to S-shaped
- How to calculate a population growth rate from births and deaths

At a glance



1. The core idea: how does the quantity change?

Model choice depends on whether the change is additive, multiplicative, or limited by the environment.

Linear growth

A quantity increases or decreases by a constant amount per time unit.

Think: +50, +50, +50 ...

Exponential growth

A quantity changes by a constant factor or percentage per time unit.

Think: ×1.05, ×1.05, ×1.05 ...

Logistic growth

Growth looks exponential at first, but limiting factors slow it and the curve levels off.

Fast test for students

- If the amount added each step stays the same, the model is linear.
- If the ratio or percent change stays the same, the model is exponential.
- If growth slows because resources run out, the model is logistic.

Identify the review-guide examples

- Town population +1,000 each year → LINEAR
- Company profit +12% each year → EXPONENTIAL
- Deer population limited by food/space → LOGISTIC

2. Linear growth

A straight-line model with a constant rate of change.

Key idea

- Add the same amount each time period.
- The graph is a straight line.
- The rate of change (slope) is constant.
- Common form: initial value + (rate)(time).

Example

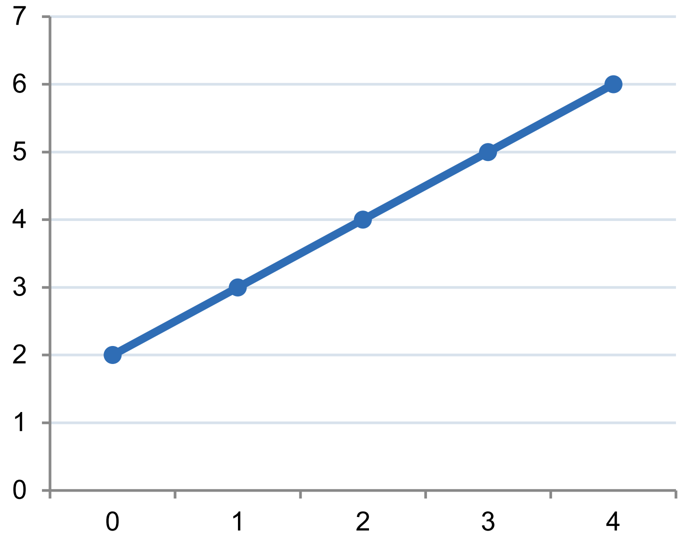
A town population increases by 1,000 people every year.

This is linear because the change is a constant amount, not a constant percent.

Model and graph

$$y = a + bt$$

Here, a is the initial value and b is the amount added each period.



Constant differences

2	
3	+1
4	+1
5	+1
6	+1

3. Exponential growth

A multiplicative model with a constant factor or percent change.

Key idea

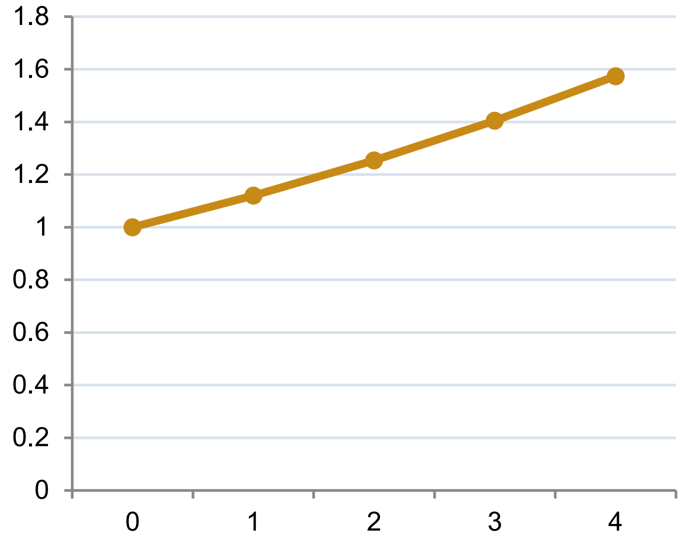
- Multiply by the same factor each period.
- Equivalent statement: increase by the same percent each period.
- The graph curves upward when the growth rate is positive.
- Common forms: initial × (factor)^time or initial × (1+r)^time.

Example

A company's profit increases by 12% each year.

Model and graph

$y = a(1 + r)^t$ If r = 0.12, then each year multiplies the amount by 1.12.



Constant factor

100	
112	×1.12
125.44	×1.12
140.49	×1.12

This is exponential because a constant percentage means repeated multiplication.

3. Exponential growth

A multiplicative model with a constant factor or percent change.

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- Multiply by the same factor each period.
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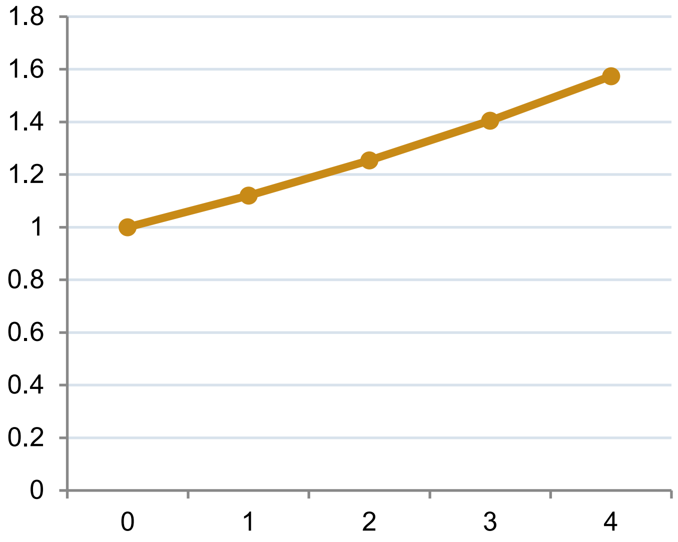

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4. Worked example: doubling spores

A mold culture starts with 50 spores and doubles every 6 hours.

Step-by-step

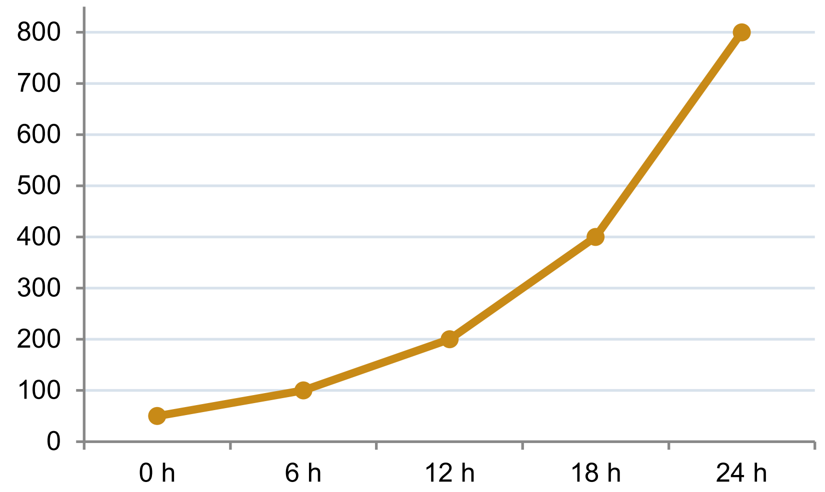
Question: How many spores after 24 hours?

- Find the number of doubling periods: 24 ÷ 6 = 4.
- Use the growth model: final = initial × factor^(number of periods).
- Substitute the values: 50 × 2^4.
- Compute: 50 × 16 = 800.

Answer: after 24 hours, the culture contains 800 spores.

$$50 \times 2^4 = 800$$

Visual pattern



Every 6 hours, the amount is multiplied by 2. The jump becomes larger because the same factor acts on a larger quantity.

5. Doubling time: estimate or exact value?

Estimate with Rule of 70 or calculate with a logarithmic formula.

Rule of 70

Use this for a quick estimate:

Doubling time ≈ 70 ÷ growth rate (%)

Example: at 5% growth, 70 ÷ 5 = 14 years.

Best when you need a mental estimate and the growth rate is in a common range.

**At 5% growth:
14 years**

Precise logarithmic formula

For exact doubling time with annual growth rate r, solve

(1+r)^t = 2

This gives t = ln(2) ÷ ln(1+r).

Use this in science, finance, or any setting that requires accuracy.

When rates are very small or very large, the estimate can drift away from the exact answer.

$$t = \ln(2) / \ln(1 + r)$$

6. Carrying capacity and logistic growth

Populations cannot grow forever because the environment imposes limits.

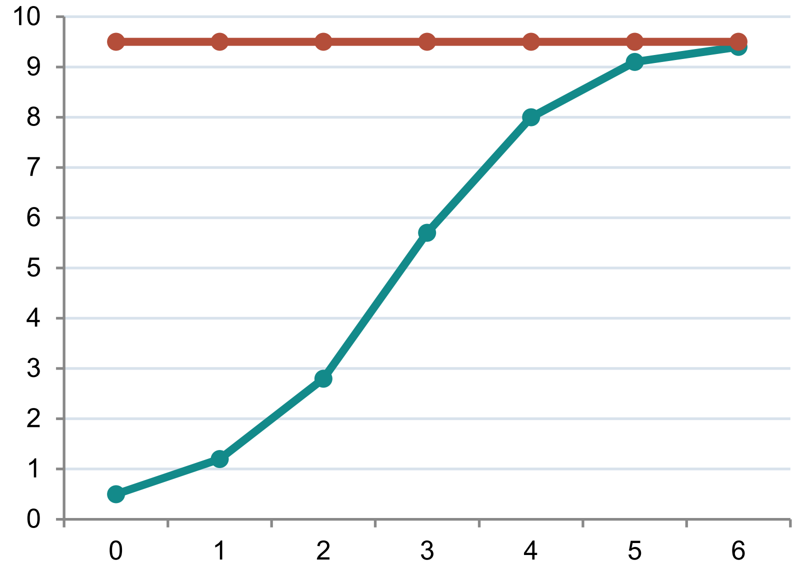
Definition

- Carrying capacity (K) is the maximum sustainable population size in a given environment.
- Growth slows as the population approaches K.
- The graph is S-shaped, not J-shaped.

Typical limiting factors

Food | Water | Space | Predators | Disease

Logistic curve

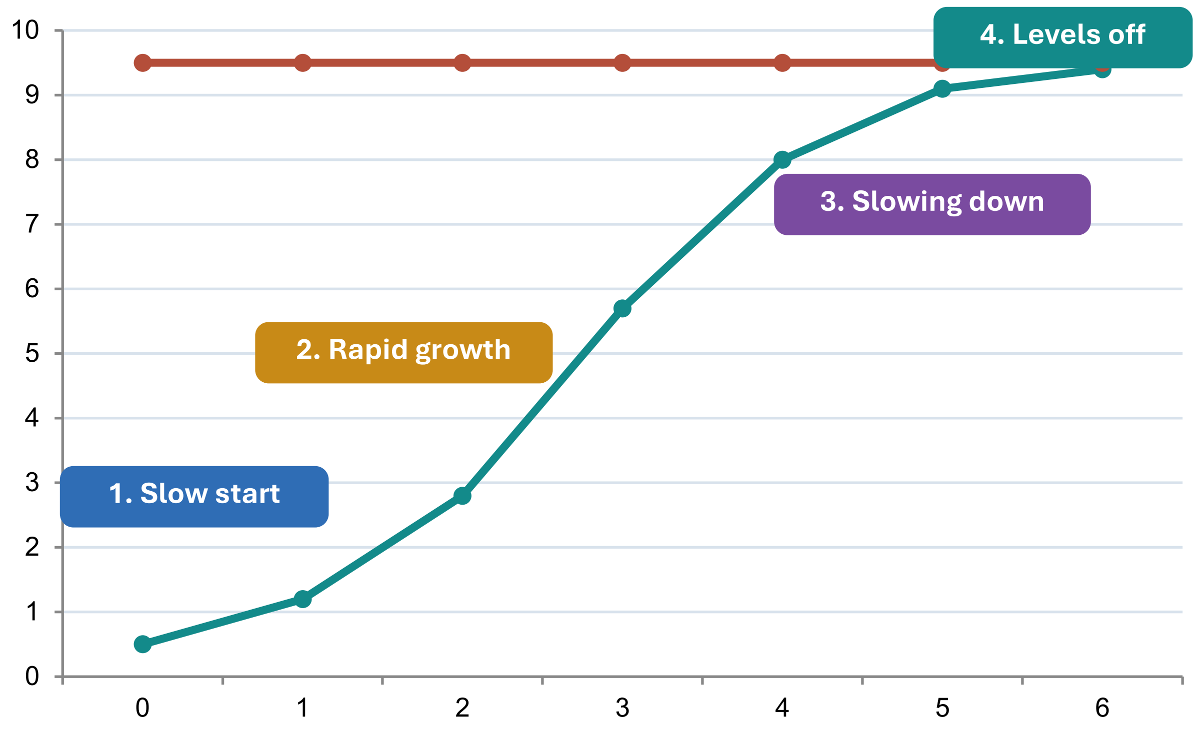


The curve begins slowly, speeds up, then slows and levels off near K.

K = carrying capacity

7. Phases of a logistic curve

Students should be able to describe what happens to the growth rate across the curve.



- Slow start: the initial population is small, so total increase is small.
- Rapid growth: the population behaves almost like exponential growth.
- Slowing down: competition for resources increases.
- Levels off: near carrying capacity, growth rate approaches zero.

8. Population growth rate from births and deaths

Example: 25 births per 1,000 people and 10 deaths per 1,000 people annually.

Computation

- Net increase per 1,000 people: 25 − 10 = 15.
- Convert to a decimal rate: 15 ÷ 1000 = 0.015.
- Convert the decimal to a percentage: 0.015 × 100% = 1.5%.

Answer: the natural population growth rate is 1.5% per year.

Why divide by 1,000?

Birth and death rates are often reported per 1,000 people.
To convert a "per 1,000" rate to a decimal, divide by 1,000.
To convert the decimal to a percent, multiply by 100.

$$(25 - 10) / 1000 = 0.015 = 1.5\%$$

9. Another exponential example: compound growth

Start with \$500 and grow by 8% each year for 3 years.

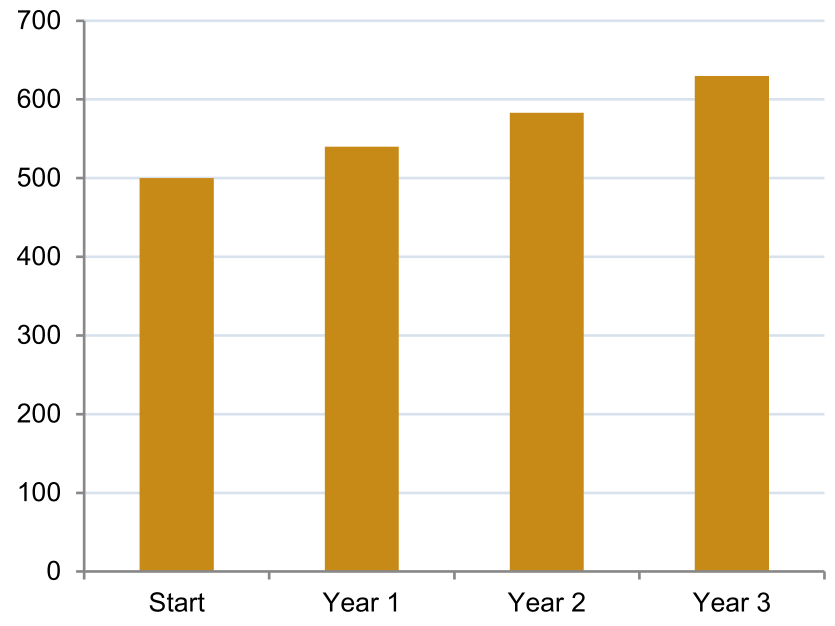
Method 1: multiply each year

Year 1: \$500 × 1.08 = \$540
Year 2: \$540 × 1.08 = \$583.20
Year 3: \$583.20 × 1.08 = \$629.856

Method 2: use the formula

$A = P(1 + r)^t$ $A = 500(1.08)^3 \approx 629.856$

Answer: about \$629.86 after 3 years.



The bars rise by larger amounts each year because the 8% is applied to a growing base.

10. Choosing the right model

Real situations often signal the model in the wording of the problem.

LINEAR

Signal words: same amount, per year, fixed raise, increases by 1,000

Examples: town population +1,000 each year; salary rising by a fixed dollar amount; water level in a cylindrical tank filled at a constant rate

Straight line graph

EXPONENTIAL

Signal words: same percent, doubles, compound interest, grows by 8%

Examples: profit +12% each year; mold spores doubling; savings with 8% growth; bacteria under ideal conditions

J-shaped curve

LOGISTIC

Signal words: limited resources, carrying capacity, levels off

Examples: deer population in a forest with limits from food, water, space, predators, and disease

S-shaped curve

11. Final study sheet

These are the ideas students should be able to recall quickly on a quiz.

Know these distinctions

- Linear: constant amount added
- Exponential: constant factor or percent
- Logistic: exponential at first, then slows near carrying capacity

Know these formulas

- Linear form: $y = a + bt$
- Exponential form: $y = a(1+r)^t$
- Doubling estimate: $70 \div$ growth rate (%)
- Exact doubling time: $\ln(2) \div \ln(1+r)$

Know these habits

- Check whether the change is additive or multiplicative
- Convert “per 1,000” to a decimal before converting to percent
- Use context to decide whether a population should level off

If students can identify the model, explain why it fits, and carry out the sample computations, they are ready for the homework and quiz.

Linear vs Exponential Models (Full Lesson)

- Complete coverage for homework mastery
- Includes concepts, formulas, examples, and practice

Key Definitions

- Linear: constant additive change
- Exponential: constant multiplicative change
- Absolute vs relative change distinction

Core Formulas

- Linear: $y = yo + rt$
- Exponential growth: $y = yo(1+r)^t$
- Exponential decay: $y = yo(1-r)^t$

Worked Example 1 (Linear)

- Initial: 1600, Rate: 642/year
- $y = 1600 + 642t$
- After 4 years → 4168

Worked Example 2 (Exponential)

- Initial: 150, Rate: 30%
- $y = 150(1.3)^t$
- After 3 months → ≈ 330

Doubling Model

- $y = yo * 2^{(t/T)}$
- Each period doubles
- Rapid growth pattern

Half-Life Model

- $y = yo*(1/2)^(t/T)$
- Used for decay
- Common in physics and biology

Rule of 70

- Doubling time ≈ 70 / r
- r is percent growth
- Valid for small rates (<15%)

Practice 1

- Population grows by 500 per year
- Initial = 2000
- Find population after 3 years

Solution 1

- Linear model
- $2000 + 500(3) = 3500$

Practice 2

- Population grows 10% yearly
- Initial = 2000
- Find population after 3 years

Solution 2

- Exponential model
- $2000(1.1)^3 \approx 2662$

Practice 3 (Doubling)

- Doubling every 5 years
- Initial = 100
- After 10 years?

Solution 3

- Two doublings
- $100 \longrightarrow 200 \longrightarrow 400$

Estimation Strategy

- Use bounds with powers
- Compare to nearby integers
- Avoid full computation when possible

Unit Conversion

- Convert time units carefully
- Example: yearly → per minute
- Always track units explicitly

Common Errors

- Percent vs decimal confusion
- Wrong formula selection
- Incorrect exponent usage

Mastery Checklist

- Identify model correctly
- Apply correct formula
- Compute accurately
- Interpret results in context