Math 114 Module 10: Normal Distribution

Prof. Volk

Spring 2025

Homework 10: 20 Questions

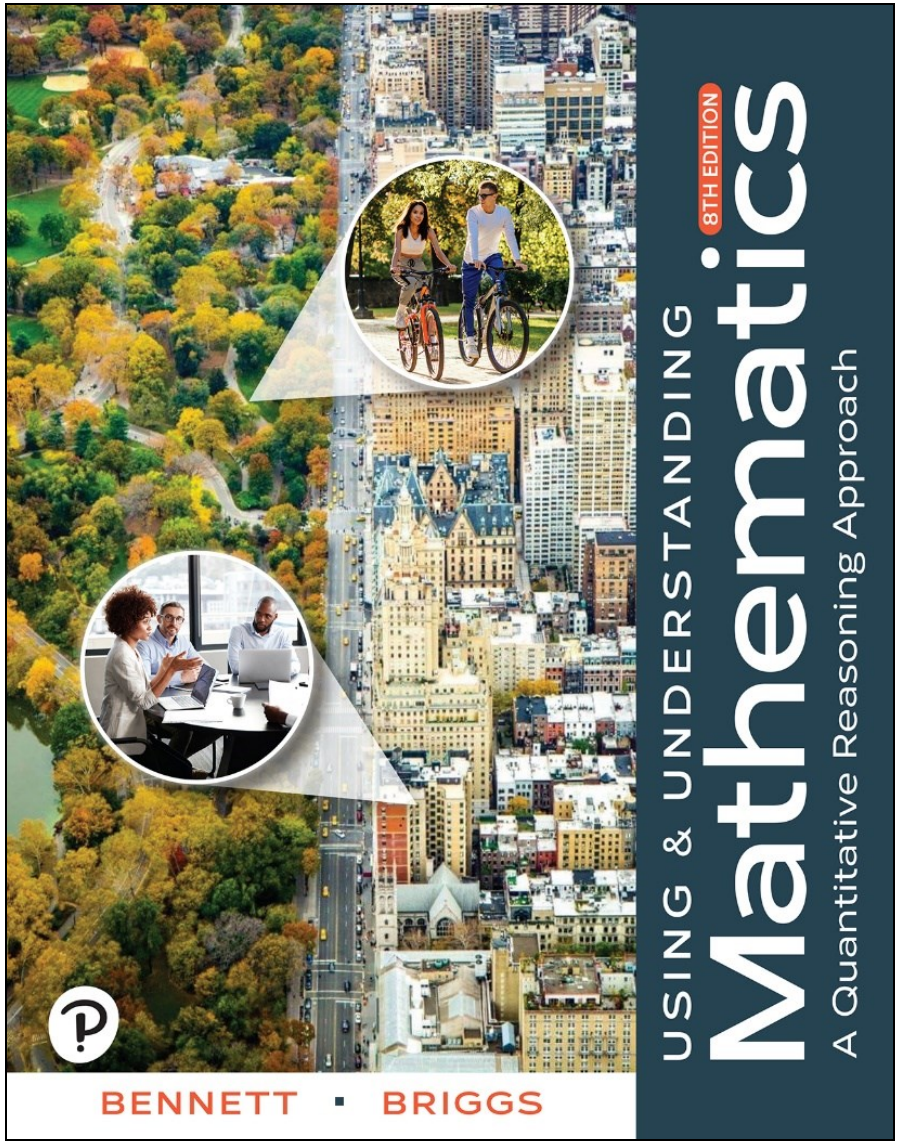
Due Sunday 11:59 pm

Questions 1 - 7	• Identify and explain characteristics of the normal distribution.
Question 8	• Interpret standard deviations on the normal distribution using the 68-95-99.7 rule.
Questions 9 - 14	• Interpret standard scores and percentiles.
Questions 15-17, 20	• Solve applications involving normal distributions and percentiles.
Questions 18-19	• Use a standard score table to find the corresponding percentiles.

Quiz 10: 10 Questions

Due Monday 11:59 pm

Question 1	• Identify and explain characteristics of the normal distribution.
Question 2	• Interpret standard scores and percentiles.
Question 3	• Determine whether distributions are normal
Questions 4-6	• Solve applications involving normal distributions and percentiles.
Questions 7-8	• Use a standard score table to find the corresponding percentiles.
Questions 9-10	• Identify and explain characteristics of the normal distribution.



This Week's Reading

Chapter 6

(Section C)

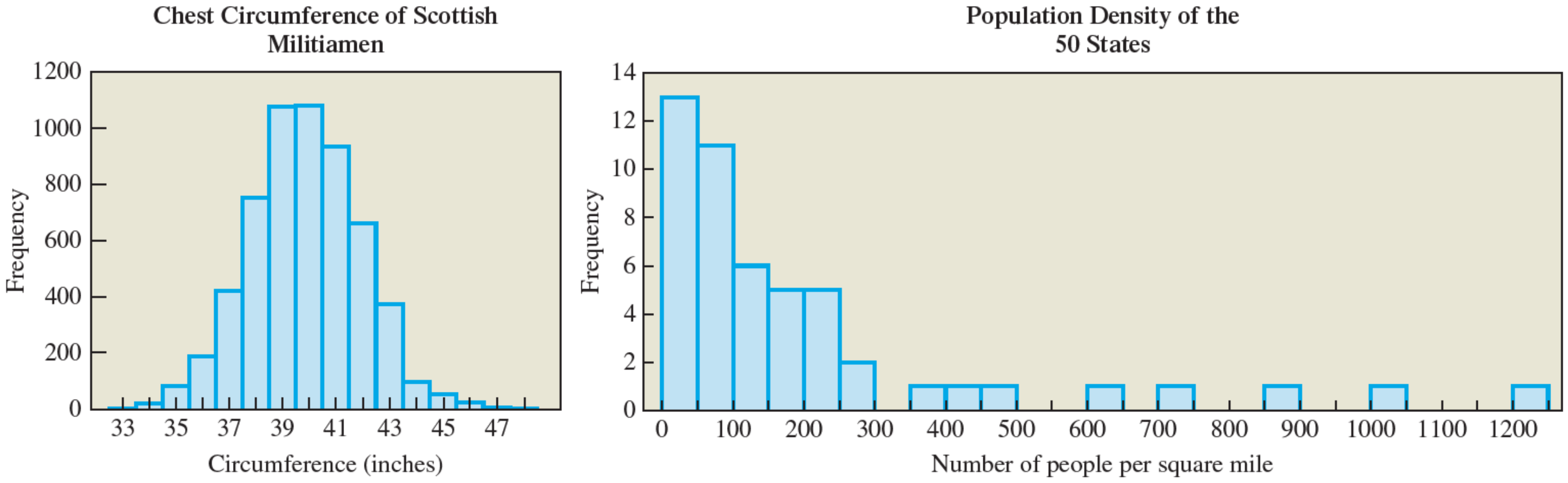
The Normal Distribution

The Normal Distribution

The **normal distribution** is a symmetric, bell-shaped distribution with a single peak. Its peak corresponds to the mean, median, and mode of the distribution. Its variation is characterized by the standard deviation of the distribution.

Example: The Normal Shape (1 of 2)

The figure shows two distributions: (a) a famous data set of the chest sizes of 5,738 Scottish militiamen collected in about 1846 and (b) the distribution of the population densities of the 50 states. Is either distribution a normal distribution? Explain.



(a) (b)

Example: The Normal Shape (2 of 2)

Solution

The distribution of chest sizes in the figure is nearly symmetric, with a mean between 39 and 40 inches. Values far from the mean are less common, giving it the bell shape of a normal distribution. The distribution in the figure on the right shows that most states have low population densities, but a few have much higher densities. This fact makes the distribution right-skewed, so it is not a normal distribution.

Conditions for a Normal Distribution

A data set satisfying the following criteria is likely to have a nearly normal distribution.

1. Most data values are clustered near the mean, giving the distribution a well-defined single peak.
2. Data values are spread evenly around the mean, making the distribution symmetric.
3. Larger deviations from the mean are increasingly rare, producing the tapering tails of the distribution.
4. Individual data values result from a combination of many different factors.

Example: Is It a Normal Distribution (1 of 2)

Which of the following variables would you expect to have a normal or nearly normal distribution?

a. Scores on a very easy test

b. Foot lengths of a random sample of adult women

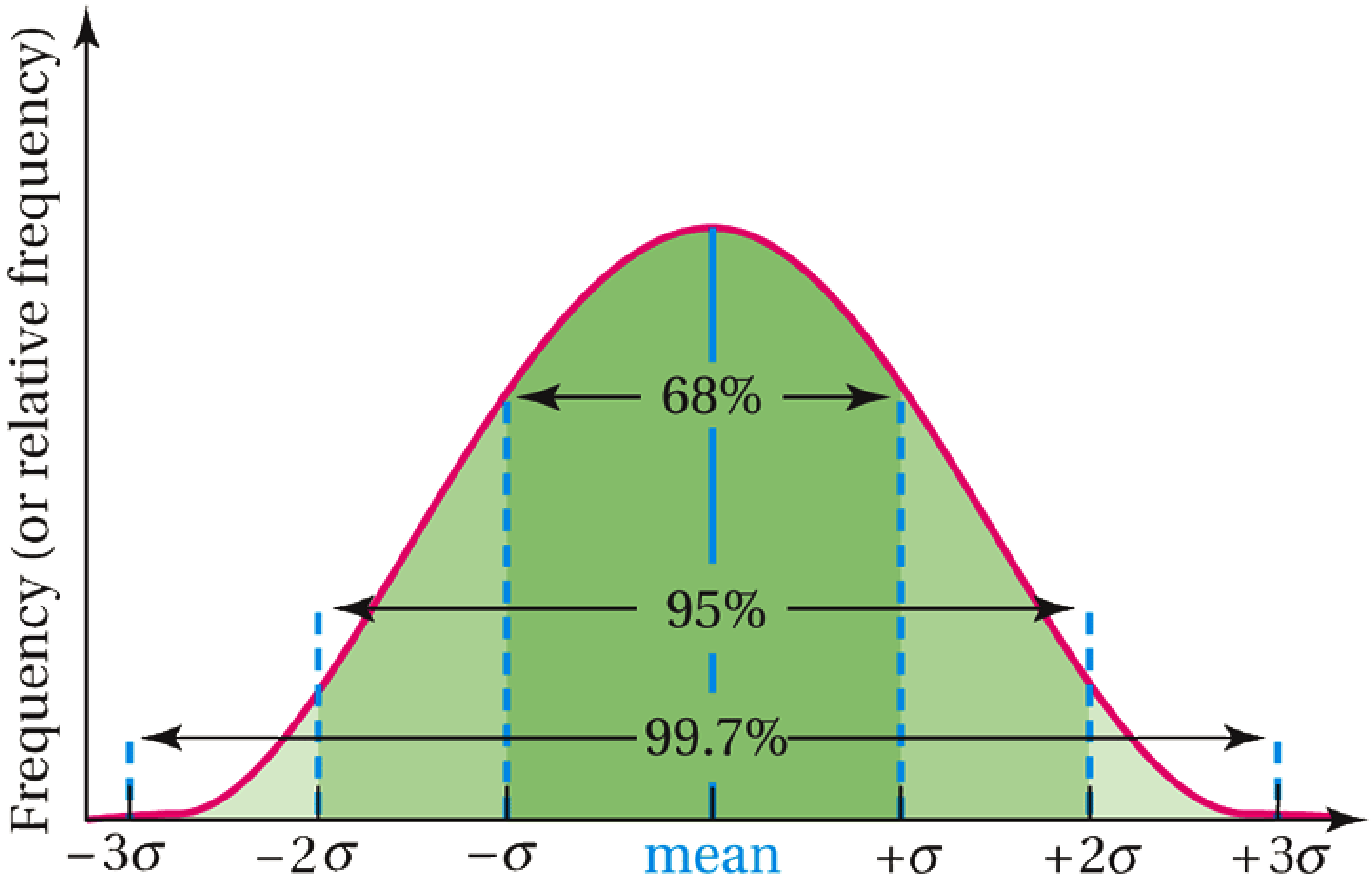
Solution

a. Tests have a maximum score (100%) that limits the size of the data values. If a test is very easy, the mean will be high and many scores will be near the maximum. The few lower scores can be spread out well below the mean. We therefore expect the distribution of scores to be left-skewed and not normal.

Example: Is It a Normal Distribution (2 of 2)

b. Foot length is a human trait determined by many genetic and environmental factors. We therefore expect lengths of women’s feet to cluster near a mean and become less common farther from the mean in both directions, giving the distribution the bell shape of a normal distribution.

The 68-95-99.7 Rule for a Normal Distribution (1 of 2)



The 68-95-99.7 Rule for a Normal Distribution (2 of 2)

About 68% (more precisely, 68.3%), or just over two-thirds, of the data points fall within 1 standard deviation of the mean.

About 95% (more precisely, 95.4%) of the data points fall within 2 standard deviations of the mean.

About 99.7% of the data points fall within 3 standard deviations of the mean.

The 68-95-99.7 rule applies to data values that are exactly 1, 2, or 3 standard deviations from the mean.

Example: Detecting Counterfeits (1 of 2)

Vending machines can be adjusted to reject coins above and below certain weights. The weights of legal U.S. quarters are normally distributed with a mean of 5.67 grams and a standard deviation of 0.0700 gram. If a vending machine is adjusted to reject quarters that weigh more than 5.81 grams and less than 5.53 grams, what percentage of legal quarters will be rejected by the machine?

Example: Detecting Counterfeits (2 of 2)

Solution

A weight of 5.81 is 0.14 gram, or 2 standard deviations, above the mean. A weight of 5.53 is 0.14 gram, or 2 standard deviations, below the mean. Therefore, by accepting only quarters within the weight range 5.53 to 5.81 grams, the machine accepts quarters that are within 2 standard deviations of the mean and rejects those that are more than 2 standard deviations from the mean. By the 68-95-99.7 rule, about 95% of legal quarters will be accepted and about 5% of legal quarters will be rejected.

Standard Scores

The number of standard deviations a data value lies above or below the mean is called its **standard score** (or *z-score*), defined by

$$z = \text{standard score} = \frac{\text{data value} - \text{mean}}{\text{standard deviation}}$$

Data Value		**Standard Score**
above the mean	$\rightarrow$	positive
below the mean	$\rightarrow$	negative

Example: Standard IQ Scores (1 of 2)

The Stanford-Binet IQ test is scaled so that scores are normally distributed with a mean of 100 and a standard deviation of 15. Find the standard scores for IQ scores of 85, 100, and 125.

Solution

We calculate the standard scores for these IQs by using the standard score formula with a mean of 100 and a standard deviation of 15.

Example: Standard IQ Scores (2 of 2)

standard score for 85 : $z = \dfrac{85 - 100}{15} = -1.00$

standard score for 100 : $z = \dfrac{100 - 100}{15} = 0.00$

standard score for 125 : $z = \dfrac{125 - 100}{15} = 1.67$

Standard Scores and Percentiles (1 of 3)

The **percentile** of a data value is the percentage of all data values in a data set that are *less than or equal to* it.

A data value that lies between two percentiles is said to lie *in* the lower percentile.

Standard Scores and Percentiles (2 of 3)

TABLE: Standard Scores and Percentiles

Z*score	Percentile	Z*score	Percentile	Z*score	Percentile	Z*score	Percentile
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72

Standard Scores and Percentiles (3 of 3)

Z*score	Percentile	Z*score	Percentile	Z*score	Percentile	Z*score	Percentile
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	−0.0	50.00	1.0	84.13	3.5	99.98

Example: Cholesterol Levels (1 of 3)

Cholesterol levels in men 18 to 24 years of age are normally distributed with a mean of 178 and a standard deviation of 41.

a. In what percentile is a 20-year-old man with a cholesterol level of 190?
b. What cholesterol level corresponds to the 90th percentile, the level at which treatment may be necessary?

Example: Cholesterol Levels (2 of 3)

Solution

a. The standard score for a cholesterol level of 190 is

$$z = \frac{\text{data value} - \text{mean}}{\text{standard deviation}} = \frac{190 - 178}{41} \approx 0.29$$

Table 6.4 shows that a standard score of 0.29 corresponds to almost the 62nd percentile.

Example: Cholesterol Levels (3 of 3)

Solution

b. The table shows that 90.32% of all data values have a standard score less than 1.3. That is, the 90th percentile is about 1.3 standard deviations above the mean. The standard deviation is 41, so 1.3 standard deviations is $1.3 \times 41 = 53.3$. We find that the 90th percentile begins at about $178 + 53 = 231$. A person with a cholesterol level above 231 may need treatment.

Notes

Score

Mean

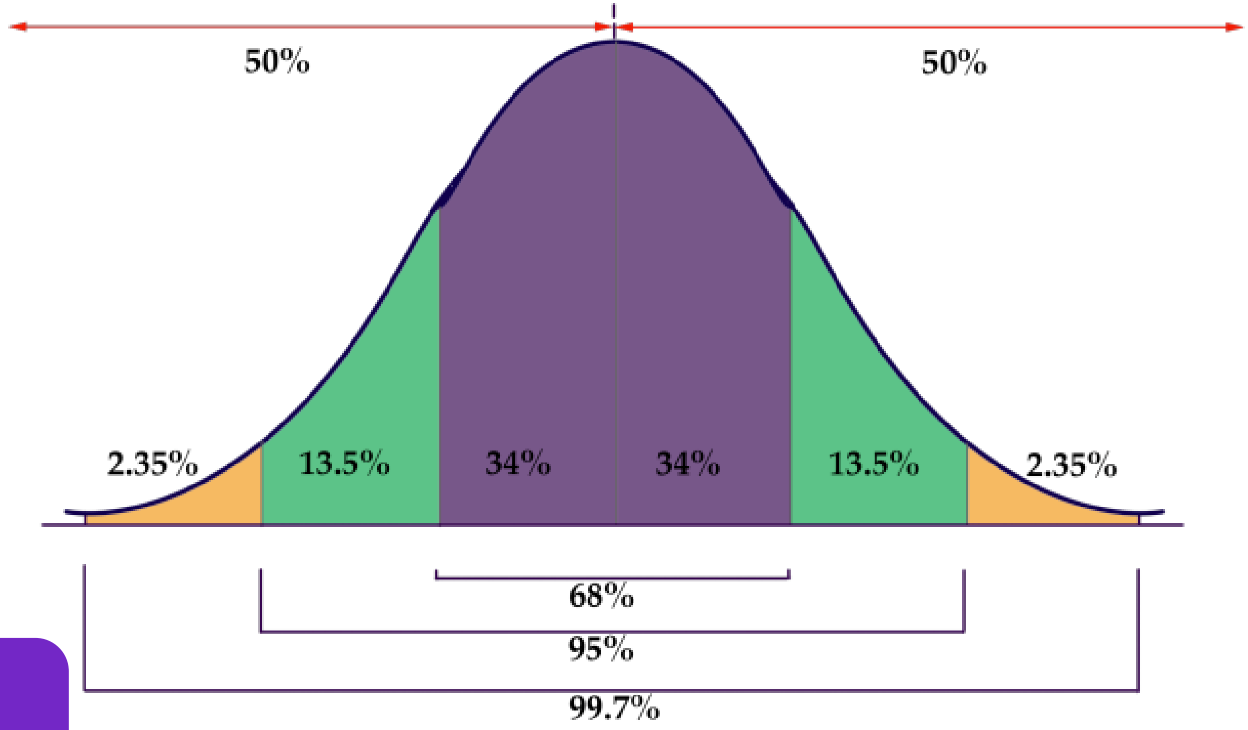
$$Z = \frac{x - \mu}{\sigma}$$

SD

Data Value

Standard Score

Percentile or Probability



The 68-95-99.7% rule

Using the 68-95-99.7% rule, we can work out the percentage of data in each section of the bell curve.

Example 1

Use the normal distribution of IQ scores, which has a mean of 100 and a standard deviation of 16, and the following table with the standard scores and percentiles for a normal distribution to find the indicated quantity.
[6] Click the icon to view the table.

Percentage of scores greater than 84 is $\boxed{\begin{array}{c}100-15.87= \\ 84.13\end{array}}$ %.

(Round to two decimal places as needed.)

below

Standard score	%	Standard score	%
−3.0	0.13	0.1	53.98
−2.5	0.62	0.5	69.15
−2	2.28	0.9	81.59
−1.5	6.68	1	84.13
−1	15.87	1.5	93.32
−0.9	18.41	2	97.72
−0.5	30.85	2.5	99.38
−0.1	46.02	3	99.87
0	50.00	3.5	99.98

Score

84 - 100

Mean

$$Z = \frac{x - \mu}{\sigma}$$

SD

16

Example 2

A set of data items is normally distributed with a mean of 20 and a standard deviation of 9. Convert 16 to a z-score.

$z_{16} = \square$

$= \text{Standardize}(16, 20, 9)$

$$Z = \frac{X - \mu}{\sigma}$$

$x = 16$

$\mu = 20$

$\sigma = 9$

$$\boxed{-0.444}$$

Example 3

Use the accompanying table of standard scores and their percentiles under the normal distribution to find the approximate standard score of the following data values. Then state the approximate number of standard deviations that the value lies above or below the mean. (Hint: Look for the percentile value that is closest to the given value and use the corresponding z-score as an approximation. If a percentile lies halfway between two percentiles in the table, take the average of the corresponding z-scores.)

a. A data value in the 60th percentile
b. A data value in the 80th percentile
c. A data value in the 17th percentile

Standard Scores and Percentiles

z-score	Percentile	z-score	Percentile	z-score	Percentile	z-score	Percentile
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98

Example 4

A set of data values is normally distributed with a mean of 40 and a standard deviation of 4. Give the standard score and approximate percentile for a data value of 43.2.

8 Click the icon to view the table of standard scores and percentiles.

The standard score for 43.2 is z = $\boxed{0.8}$.

(Do not round until the final answer. Then round to the nearest hundredth as needed.)

The data value 43.2 occurs at about the $\boxed{79}$ th percentile.

(Round to the nearest whole number as needed.)

Score 43.2- 40 Mean

$$Z = \frac{x - \mu}{\sigma}$$

SD 4

Standard Scores and Percentiles

z-score	Percentile	z-score	Percentile	z-score	Percentile	z-score	Percentile
− 3.5	0.02	− 1.0	15.87	0.0	50.00	1.1	86.43
− 3.0	0.13	− 0.95	17.11	0.05	51.99	1.2	88.49
− 2.9	0.19	− 0.90	18.41	0.10	53.98	1.3	90.32
− 2.8	0.26	− 0.85	19.77	0.15	55.96	1.4	91.92
− 2.7	0.35	− 0.80	21.19	0.20	57.93	1.5	93.32
− 2.6	0.47	− 0.75	22.66	0.25	59.87	1.6	94.52
− 2.5	0.62	− 0.70	24.20	0.30	61.79	1.7	95.54
− 2.4	0.82	− 0.65	25.78	0.35	63.68	1.8	96.41
− 2.3	1.07	− 0.60	27.43	0.40	65.54	1.9	97.13
− 2.2	1.39	− 0.55	29.12	0.45	67.36	2.0	97.72
− 2.1	1.79	− 0.50	30.85	0.50	69.15	2.1	98.21
− 2.0	2.28	− 0.45	32.64	0.55	70.88	2.2	98.61
− 1.9	2.87	− 0.40	34.46	0.60	72.57	2.3	98.93
− 1.8	3.59	− 0.35	36.32	0.65	74.22	2.4	99.18
− 1.7	4.46	− 0.30	38.21	0.70	75.80	2.5	99.38
− 1.6	5.48	− 0.25	40.13	0.75	77.34	2.6	99.53
− 1.5	6.68	− 0.20	42.07	0.80	78.81	2.7	99.65
− 1.4	8.08	− 0.15	44.04	0.85	80.23	2.8	99.74
− 1.3	9.68	− 0.10	46.02	0.90	81.59	2.9	99.81
− 1.2	11.51	− 0.05	48.01	0.95	82.89	3.0	99.87
− 1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98

Example 5

The scores for a certain test of intelligence are normally distributed with mean 130 and standard deviation 13. Find the 90th percentile of these scores.

[9] Click the icon to view the table of standard scores and percentiles.

The 90th percentile is __________. (Round to the nearest whole number as needed.)

$$1.3 = \frac{x - 130}{13}$$

Score

Mean

$$Z = \frac{x - \mu}{\sigma}$$

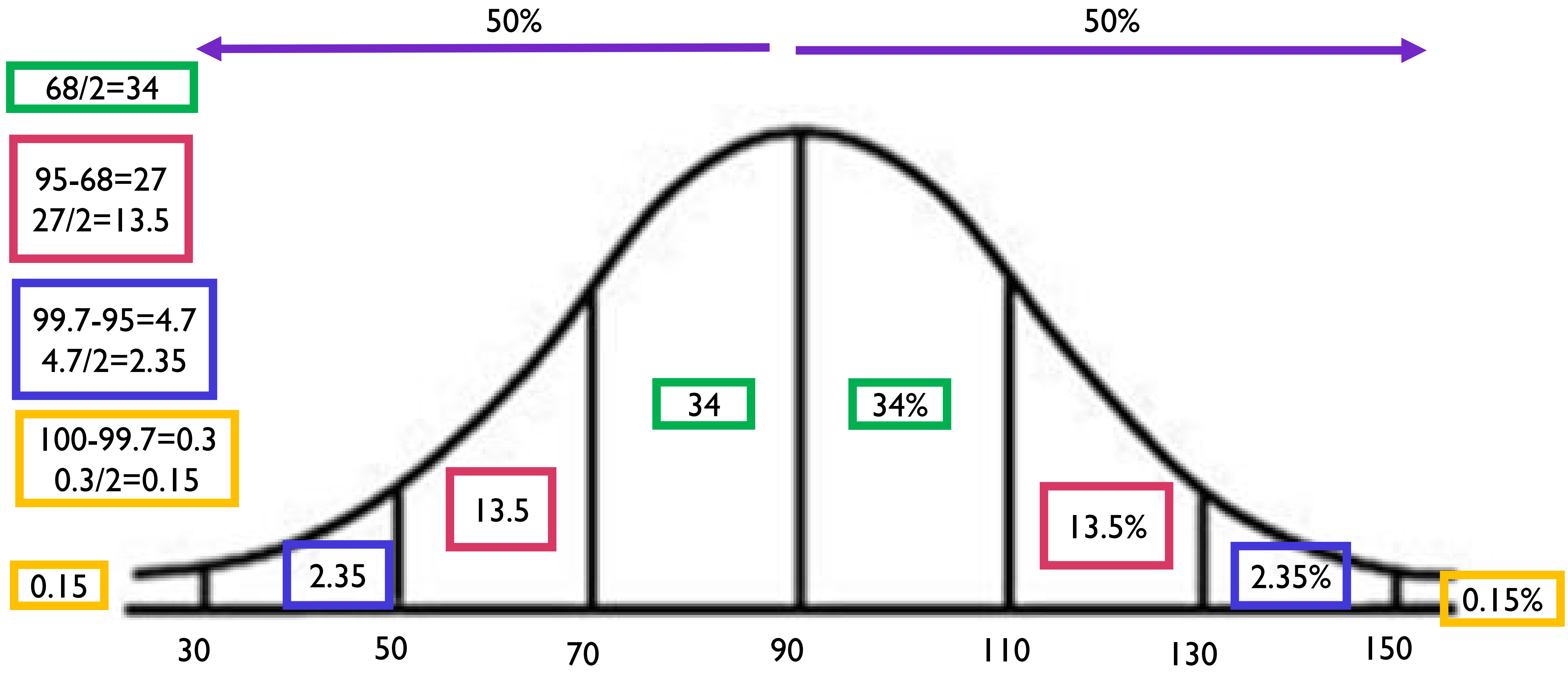
SD

Standard Scores and Percentiles

z-score	Percentile	z-score	Percentile	z-score	Percentile	z-score	Percentile
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98

Example 6

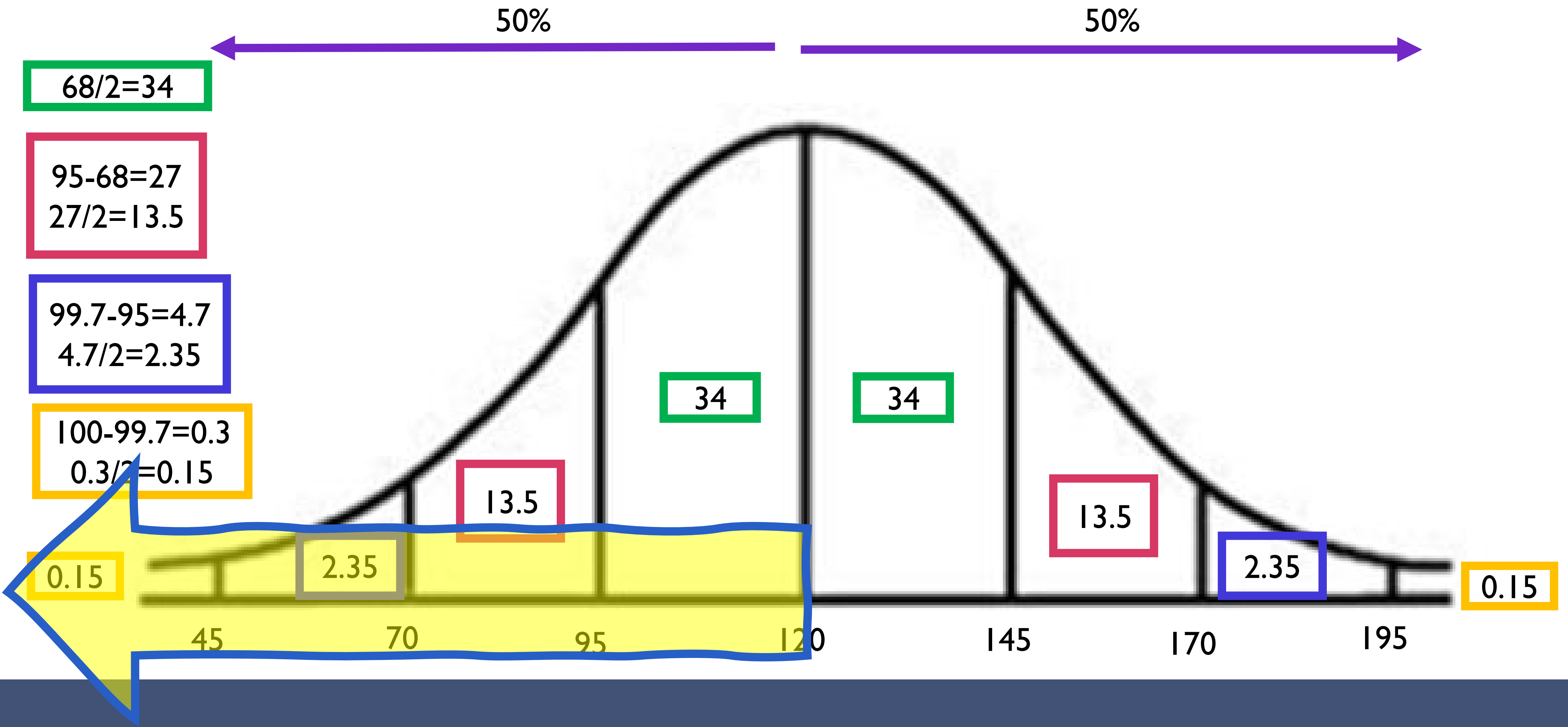
Assume that a set of test scores is normally distributed with a mean of 120 and a standard deviation of 25. Use the 68-95-99.7 rule to find the following quantities.



Example 6

a. The percentage of scores less than 120 is ____50%____ %.

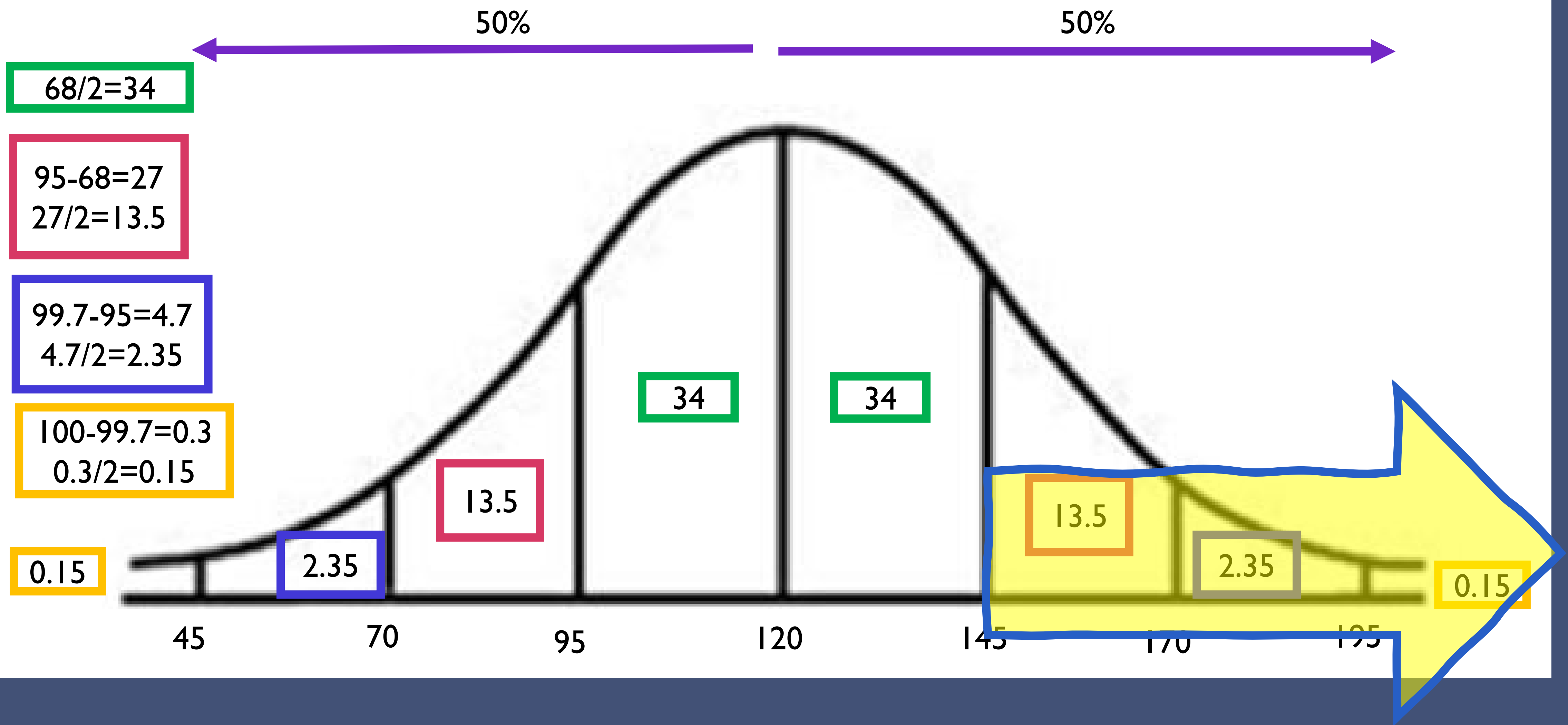
(Round to one decimal place as needed.)



Example 6

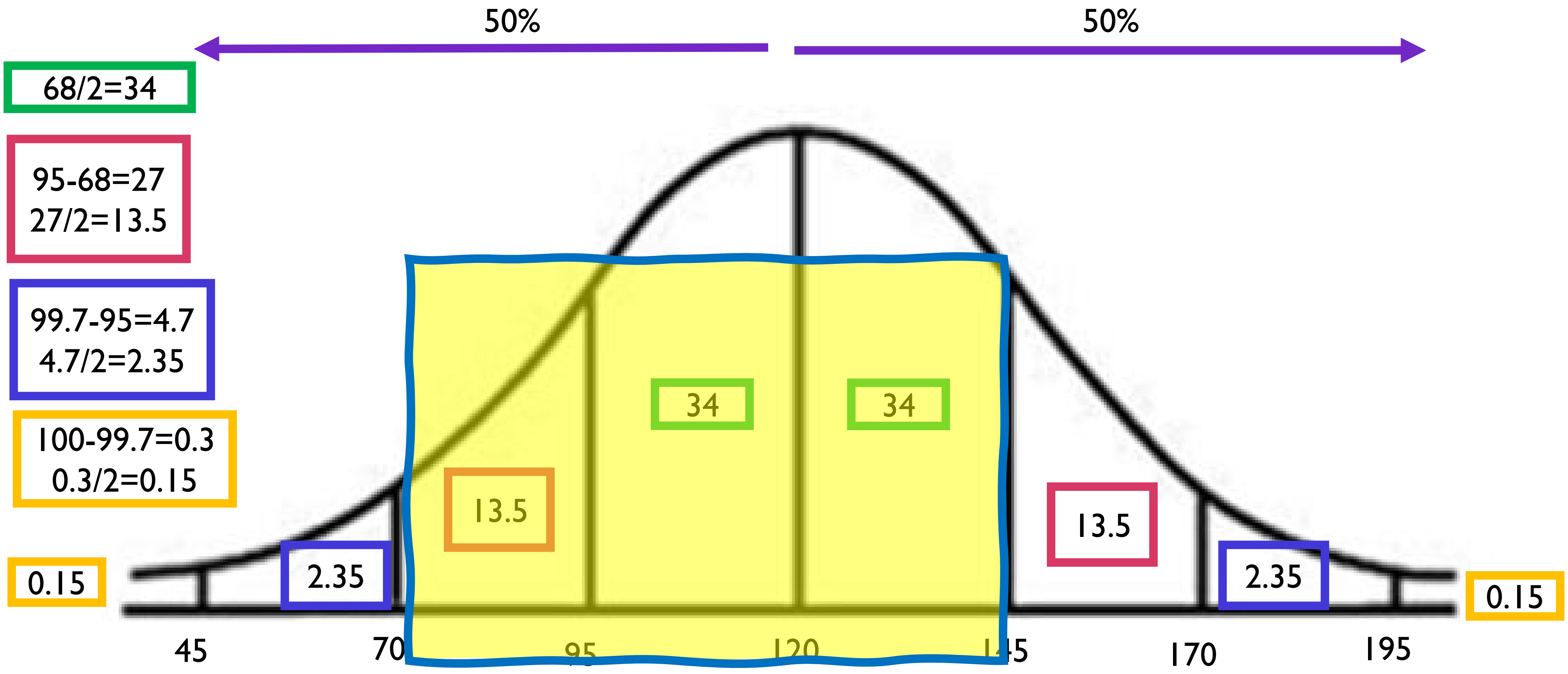
b. The percentage of scores greater than 145 is 16% %.

(Round to one decimal place as needed.)



Example 6

c. The percentage of scores between 70 and 145 is 81.5 %.
(Round to one decimal place as needed.)



Example 7

a. A data value in the 80th percentile
b. A data value in the 60th percentile
c. A data value in the 54th percentile

z-score	Percentile
0.0	50.00
0.05	51.99
0.10	53.98
0.15	55.96
0.20	57.93
0.25	59.87
0.30	61.79
0.35	63.68
0.40	65.54
0.45	67.36
0.50	69.15
0.55	70.88
0.60	72.57
0.65	74.22
0.70	75.80
0.75	77.34
0.80	78.81
0.85	80.23
0.90	81.59
0.95	82.89
1.0	84.13

a) $z = .85$

b) $z = 0.25$

c) $z = 0.10$

Example 8

Scores on the quantitative portion of an exam have a mean of 572 and a standard deviation of 148. Assume the scores are normally distributed. What percentage of students taking the quantitative exam score above 609?

$\mu = 572 \quad \sigma = 148 \quad x = 609$

$$z = \frac{609 - 572}{148} = 0.25$$

Percentile → 59.87

% below

100 − 59.87 → $\boxed{40\% \text{ above}}$

Standard Scores and Percentiles

z-score	Percentile	z-score	Percentile
− 1.0	15.87	0.0	50.00
− 0.95	17.11	0.05	51.99
− 0.90	18.41	0.10	53.98
− 0.85	19.77	0.15	55.96
− 0.80	21.19	0.20	57.93
− 0.75	22.66	0.25	59.87
− 0.70	24.20	0.30	61.79
− 0.65	25.78	0.35	63.68
− 0.60	27.43	0.40	65.54
− 0.55	29.12	0.45	67.36
− 0.50	30.85	0.50	69.15
− 0.45	32.64	0.55	70.88
− 0.40	34.46	0.60	72.57
− 0.35	36.32	0.65	74.22
− 0.30	38.21	0.70	75.80
− 0.25	40.13	0.75	77.34
− 0.20	42.07	0.80	78.81
− 0.15	44.04	0.85	80.23
− 0.10	46.02	0.90	81.59
− 0.05	48.01	0.95	82.89
0.0	50.00	1.0	84.13

POINTS EARNED THROUGH MODULE 9

100%	560
90%	504
80%	448
70%	392

Notes

Score

Mean

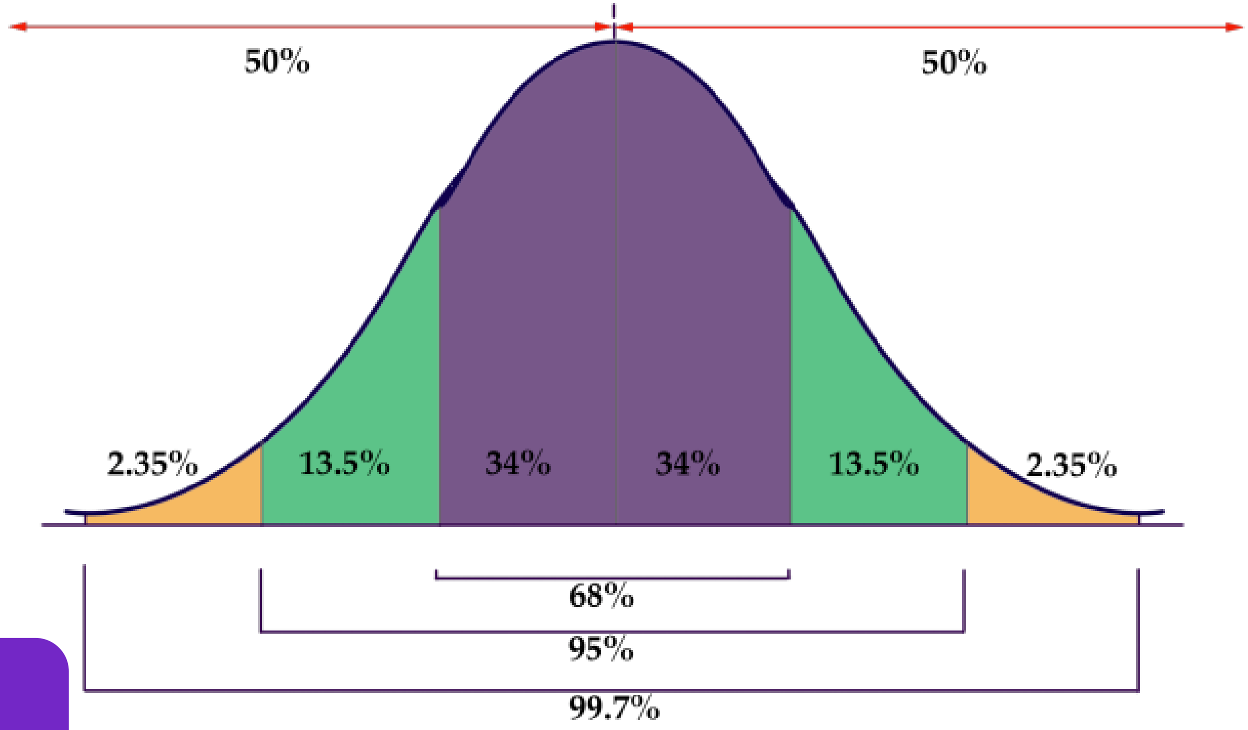
$$Z = \frac{x - \mu}{\sigma}$$

SD

Data Value

Standard Score

Percentile or Probability



The 68-95-99.7% rule

Using the 68-95-99.7% rule, we can work out the percentage of data in each section of the bell curve.

Example

Scores on the quantitative portion of an exam have a mean of 589 and a standard deviation of 157. Assume the scores are normally distributed. What percentage of students taking the quantitative exam score below 613?

[11] Click the icon to view the table of standard scores and percentiles.

What percentage of students taking the quantitative exam score below 613?

56 % (Round to the nearest whole number as needed.)

What percentage of students taking the quantitative exam score Above 600?

100-53= 47 % (Round to the nearest whole number as needed.)

What percentage of students Between 600 and 700?

76-53 = 23 % (Round to the nearest whole number as needed.)

$Z = .07 \rightarrow 53$

$Z = .70 \rightarrow 76$

Standard Scores and Percentiles

z-score	Percentile	z-score	Percentile	z-score	Percentile	z-score	Percentile
−3.5	0.02	−1.0	15.87	0.0	50.00	1.1	86.43
−3.0	0.13	−0.95	17.11	0.05	51.99	1.2	88.49
−2.9	0.19	−0.90	18.41	0.10	53.98	1.3	90.32
−2.8	0.26	−0.85	19.77	0.15	55.96	1.4	91.92
−2.7	0.35	−0.80	21.19	0.20	57.93	1.5	93.32
−2.6	0.47	−0.75	22.66	0.25	59.87	1.6	94.52
−2.5	0.62	−0.70	24.20	0.30	61.79	1.7	95.54
−2.4	0.82	−0.65	25.78	0.35	63.68	1.8	96.41
−2.3	1.07	−0.60	27.43	0.40	65.54	1.9	97.13
−2.2	1.39	−0.55	29.12	0.45	67.36	2.0	97.72
−2.1	1.79	−0.50	30.85	0.50	69.15	2.1	98.21
−2.0	2.28	−0.45	32.64	0.55	70.88	2.2	98.61
−1.9	2.87	−0.40	34.46	0.60	72.57	2.3	98.93
−1.8	3.59	−0.35	36.32	0.65	74.22	2.4	99.18
−1.7	4.46	−0.30	38.21	0.70	75.80	2.5	99.38
−1.6	5.48	−0.25	40.13	0.75	77.34	2.6	99.53
−1.5	6.68	−0.20	42.07	0.80	78.81	2.7	99.65
−1.4	8.08	−0.15	44.04	0.85	80.23	2.8	99.74
−1.3	9.68	−0.10	46.02	0.90	81.59	2.9	99.81
−1.2	11.51	−0.05	48.01	0.95	82.89	3.0	99.87
−1.1	13.57	0.0	50.00	1.0	84.13	3.5	99.98

Exam results for 100 students are given below. For the given exam grades, briefly describe the shape and variation of the distribution.

median = 36, mean = 86, low score = 17, high score = 96

The distribution is [▼] and has [▼] variation.

- Right Skewed
- High Variation

Assume that the prices of solar panels are normally distributed with a mean of \$2.10 per watt and a standard deviation of \$0.25 per watt. Complete parts a through c.

Click the icon to view the standard scores and percentiles for a normal distribution.

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a. What percentage of solar panels cost less than \$2.10 per watt?

50 % of solar panels cost less than \$2.10 per watt.
(Type an integer or a decimal.)

b. What percentage of solar panels cost more than \$1.70 per watt?

94.52 % of solar panels cost more than \$1.70 per watt.
(Type an integer or a decimal.)

c. Suppose you paid less for your solar panels than 60% of all buyers. About what was the price you paid in dollars per watt?

You paid about \$ 2.88 per watt.
(Round to two decimal places as needed.)

QUESTIONS?